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Transportation Infrastructure Investment in the Spanish Regions

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Abstract: In a previous article by Barreda et al. (2002) we derived, using a monopolistic competition model, the optimal sharing of a limited infrastructure investment budget between two regions, when taking into account the transportation cost reducing features of the infrastructure investment. In the present paper we generalize the optimality conditions for n regions and we obtain that the sharing of the budget is proportional to the combined effect of the population density and the scarcity of the already existing infrastructure. In order to validate these theoretical results, we proceed to test empirically if they are confirmed in the particular case of the Spanish regions. Particularly we analyze real data concerning transportation infrastructure investment costs, existing transportation infrastructure level, population density, and number of firms for 18 Spanish regions. By using regression analysis and non-parametric correlations and tests we check if the normative optimality conditions are positively fulfilled.

1. Introduction

We use the well known monopolistic competition framework[2], were the disutility to the consumer is caused by the price paid and the value lost from not consuming his ideal variety, to make a partial equilibrium analysis of welfare in a multi-region market with differential population density and transportation infrastructure conditions among regions.

In this framework the social costs, which typically have to be taken into account in order to compute social welfare, arise from two sources: (1) the sunk cost paid by firms to enter the market and (2) the transportation costs incurred from not consuming the ideal variety. Our approach to these two sources of welfare reduction will be non-standard in this study given that we consider that the impact of both of them should not necessarily have the same weight in the social cost function. This is because we interpret that fixed entry costs (for instance machinery) and transportation costs are of different nature so they can have different consequences on welfare reduction.

We interpret the framework in its spatial version, in which transportation costs can be seen as such (the other interpretation would be disutility derived from not consuming the most preferred variety) and we introduce a costly transportation cost reducing investment which has to be made

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[2] The main conclusion of these models in terms of social welfare is that too many firms/varieties enter the market in the case of a free entry long run equilibrium with respect to the social optimum. See Salop (1979) and Dixit-Stiglitz (1977).

necessarily by the individual firms and the public sector. This view relates to a different strand in the literature of monopolistic competition models[3]: models of differentiated product markets with endogenous transportation costs. They have been used to study situations in which firms build an infrastructure which facilitates the access of the potential customers. These models identify reducing the unit transportation costs to designing more *general purpose* products. Normally the reduction is reflected only on increased unit production costs and the state does not intervene in this reduction.

If we interpret the model in terms of vertical differentiation in the style of Dos Santos and Thisse (1996) we could say that a reduction in the firm specific transportation cost parameter could be seen as an increase in the firm's product quality.

We have opted for a modification of this framework assuming that investing in transportation infrastructure is both *costly* and *necessary* and must be supported by *both* private and public investment. We introduce a stage which precedes the usual entry/firm investment/price competition structure. In this first stage, the policy maker decides on the level of the investment in a public infrastructure which will be used by firms at an endogenously determined sunk cost.

The main contribution of the paper concerns costly government intervention through public investment in a transportation cost reducing infrastructure in a multi-region setting.

In fact, we consider that firms invest in installing or improving the infrastructure which is required for the transportation of economic goods from the place of production to the place of consumption. Both the state and private investors are involved in such an effort. For example, a highway may be the result of public investment, but firms have to incur costs (own or rented trucks) if they want to use the highway. Communication networks could be another example.

Our theoretical results differ from the usual variety proliferation as compared to the social optimum result because the relationship between the optimal and the equilibrium number of firms in the long run may vary depending on the relative weights of the components of the social cost function. Our results also indicate that, in the endogenous entry costs setting, a government policy addressed to entry control is ineffective in terms of rising social welfare. Instead, public investment in transportation cost reducing infrastructure is found to play an important role as a policy instrument by a social planner who acts as a leader with respect to the rest of the agents in the market. We also find that the market equilibrium is such that private investment in transportation infrastructure depends negatively on public investment. This property concerning strategic substitutability between private

[3] Hendel and Neiva (1997), Weitzman (1994), Von Ungern-Sternberg (1988), and Grossman and Shapiro (1984), are the most representative examples of the aforementioned literature.

and public investment makes public investment an effective policy instrument which can be used to minimise social costs.

Finally, another strand in the literature related to international industrial location also considers topics which can be addressed by means of our framework. That is the case of Martin and Rogers (1995) who study the incentives for firms to relocate in a given region depending on its publicly financed domestic infrastructure. Along a different line, Yamano and Ohkawara (2000) study the trade-off between efficiency and equity the central authority is faced with when deciding its investment in a developed, or in a more depressed region. Coughlin and Segev (2000) find that higher levels of economic size and transportation infrastructure are associated with a larger number of new foreign-owned plants being opened in a region in the United States.

We extend our framework to the case of endogenous entry costs and *n* regions with different characteristics. Our results show that there will be incentives for more firms to locate in a region with higher population density, or with worse existing infrastructure, while public investment in infrastructure will also be higher in this kind of region.

In a second phase of the study we have confronted the theoretical results of our framework to real statistical data for 18 Spanish regions. We obtain that a worse existing infrastructure has the positive expected influence in the sharing of the budget but the population density does not have a significant influence. Surprisingly, or maybe not, the number of parliamentary seats to be obtained from a given region has a significant positive impact on the sharing of the budget.

The article is organised as follows: Section 2 presents the basic framework which we will apply in the subsequent sections. Section 3 studies public investment optimality and its implications, in the case of endogenous entry costs. Section 4 deals with optimal constrained public investment in *n* different regions. Section 5 presents the empirical results and Section 6 concludes.

2. Framework

Consider the following version of the monopolistic competition model in its spatial form, as proposed by Salop (1979). Let *n* firms be equidistantly located around a unit periphery circle. A continuum of consumers is uniformly distributed around the circle with density equal to *d*. Each one of them is willing to buy one unit of the good from the firm whose generalised price (price plus transportation costs) at the consumer location is lower, unless the consumer's surplus were negative, in which case zero consumption would be preferred to consuming one unit.

We summarise the preceding assumptions stating that a consumer *j* purchasing a unit of the good at firm *i* maximises her utility U_{ji} as long as $U_{ji} \geq 0$ and

$$U_{ji} \geq U_{jh} \Rightarrow R - p_i - T_i(x_{ji}) \geq R - p_h - T_h(x_{jh}), \tag{1}$$

where firm *h* is firm *i*'s adjacent firm as we move (anti-)clockwise on the circle ($h \in \{i-1, i+1\}$) and p_i denotes the price charged by firm *i*. We will consider that equilibrium prices charged by the firms are low enough, or (which is equivalent) that the income of consumers - denoted by *R* in expression (1) - is high enough[4] for each one of them to buy a unit of the good from the firm whose generalised price at the consumer location is lower. Then, the market is fully covered by the sales of the firms. We will also consider that marginal production costs are *0*.

Transportation costs, paid by consumer *j* buying from firm $i \in \{1,2,...n\}$, are a linear function of the distance *x* between the locations of production (where *i* is located) and consumption (where *j* is located) of the good, respectively. This is expressed by:

$$T_i\left(x_{ji}\right) = \tau_i \cdot x_{ji} = \frac{t}{k_i} \cdot x_{ji} = \frac{w}{k_i \cdot I} \cdot x_{ji}, \tag{2}$$

where k_i and *I* are, respectively, the levels of *individual* (firm-specific) and *public* investments in the aforementioned infrastructure. Parameter *w* can be seen as the difficulty of the geographical conditions prevailing in the region or the bad state of the already existing infrastructure. The product of private and public investment in the denominator of the firm-specific unit transportation cost coefficient, τ_i, implies a positive interaction between private and public investment in the transportation cost-saving capacity of the infrastructure.

Given that consumers are uniformly distributed around the circle with a constant density *d*, firm *i*'s demand coincides with the size of the segment whose population buys from the firm multiplied by *d*. Let p_i, p_{i+1}, p_{i-1} be, respectively, the prices of firm *i*, and the firm $i+1$ ($i-1$), which is the first as we move clockwise (anti-clockwise) from it. Then, given an equidistant arrangement of firms, there will be a consumer at a distance x_i $\left(\frac{1}{n} - x_{i-1}\right)$ as we move clockwise (anti-clockwise) on the circle from firm *i*, who will be indifferent between buying from firm *i* and buying from firm $i+1$ $(i-1)$.

[4] In the Appendix, we define the exact expression for this restriction in terms of the parameters of the model.

In fact, using these two locations as the extremes of the segment supplied by firm *i*, we can write firm *i*'s demand: $q_i = d \cdot \left(x_i + \left(\frac{1}{n} - x_{i-1} \right) \right)$.

Given an equidistant arrangement of the firms, and following (1) as equality: $p_i + \tau_i \cdot x_{ji} = p_{i+1} + \tau_{i+1} \cdot \left(\frac{1}{n} - x_{ji} \right)$, we can obtain the distance from firm *i* of the indifferent consumer between firm *i* and $i+1$:

$$x_i = \frac{p_{i+1} - p_i}{\tau_i + \tau_{i+1}} + \frac{\tau_{i+1}}{n(\tau_i + \tau_{i+1})}. \tag{3}$$

So, firm *i*'s demand will be:

$$q_i = d \cdot \left(\frac{p_{i+1} - p_i}{\tau_i + \tau_{i+1}} + \frac{\tau_{i+1}}{n(\tau_i + \tau_{i+1})} + \frac{p_{i-1} - p_i}{\tau_i + \tau_{i-1}} + \frac{\tau_{i-1}}{n(\tau_i + \tau_{i-1})} \right). \tag{4}$$

This expression indicates that unit transportation costs paid by firm *i*'s clients have an unambiguously negative effect on the firm's demand[5].

It can also be concluded that the effect of unit transportation costs paid by rival firms' clients have a positive effect on firm *i*'s demand, unless rival prices are much higher than firm *i*'s own price and the number of firms is sufficiently high[6].

Finally, the effect of price differences on firm *i*'s demand depends negatively on the sum of firm-specific unit transportation cost coefficients[7].

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5 Observe that, as long as the firm's price is not too much higher than the price charged by the firm's adjacent rivals in the presence of a sufficiently low number of firms for firm *i* to have a positive share, the following conditions are satisfied: $\frac{\partial x_i}{\partial \tau_i} = \frac{n(p_i - p_{i+1}) - \tau_{i+1}}{n(\tau_i + \tau_{i+1})^2} < 0$, given that a positive market share is guaranteed for firm *i* if $p_i - p_{i+1} < \frac{\tau_{i+1}}{n}$.

6 Observe that, as long as the price charged by the rival firm is not too much higher than firm *i*'s own price and with a sufficiently low number of firms in the market, explicitly if $p_{i+1} - p_i < \frac{\tau_i}{n}$, then $\frac{\partial x_i}{\partial \tau_{i+1}} = \frac{n(p_i - p_{i+1}) + \tau_i}{n(\tau_i + \tau_{i+1})^2} > 0.$

7 Note that: $\frac{\partial x_i}{\partial (p_{i+1} - p_i)} = \frac{n(p_{i+1} - p_i) + \tau_{i+1}}{n(p_{i+1} - p_i)(\tau_i + \tau_{i+1})}$.

We will now proceed to modify this basic framework applying it to the case of endogenous entry costs in the case of *n* regions with different natural and population characteristics.

3. Endogenous Entry Costs

Let us consider an endogenous entry cost, the cost k_i faced by firm *i* in order to decrease its transportation cost parameter τ_i. We substitute τ_i with $\frac{t}{k_i}$ in firm *i*'s demand (4) and we get:

$$q_i = \frac{d}{n \cdot t}\left(\frac{n(p_{i-1} - p_i) + \frac{t}{k_{i-1}}}{\left(\frac{1}{k_{i-1}} + \frac{1}{k_i}\right)} + \frac{n(p_{i+1} - p_i) + \frac{t}{k_{i+1}}}{\left(\frac{1}{k_{i+1}} + \frac{1}{k_i}\right)}\right). \tag{5}$$

We can now write individual profits:

$$\Pi_i = \frac{p_i \cdot d}{n \cdot t}\left(\frac{n(p_{i-1} - p_i) + \frac{t}{k_{i-1}}}{\left(\frac{1}{k_{i-1}} + \frac{1}{k_i}\right)} + \frac{n(p_{i+1} - p_i) + \frac{t}{k_{i+1}}}{\left(\frac{1}{k_{i+1}} + \frac{1}{k_i}\right)}\right) - k_i. \tag{6}$$

From the first order conditions for maximisation of the profit function above with respect to p_i, we obtain the best price-response[8] of firm *i*. Then, $\frac{\partial \Pi_i}{\partial p_i} = 0 \Rightarrow p_i\left(p_{i+1}, p_{i-1}\right) =$

$$p_i = \frac{n\left(k_{i-1}(k_i + k_{i+1})p_{i-1} + k_{i+1}(k_i + k_{i-1})p_{i+1}\right) + t(k_{i-1} + k_{i+1} + 2k_i)}{2n(k_{i-1}k_i + 2k_{i-1}k_{i+1} + k_{i+1}k_i)}. \tag{7}$$

This expression of firm *i*'s reaction function implies a system of *n* equations with *n* unknown variables, which should be solved simultaneously together with the system of the following first order conditions satisfied by equilibrium entry costs: $\frac{\partial \Pi_i}{\partial k_i} = 0 \Rightarrow$

$$\frac{d \cdot p_i}{n \cdot t} \cdot \left(\frac{k_{i-1}\left(k_{i-1}n(p_{i-1} - p_i) + t\right)}{(k_i + k_{i-1})^2} + \frac{k_{i+1}\left(k_{i+1}n(p_{i+1} - p_i) + t\right)}{(k_i + k_{i+1})^2}\right) = 1, \tag{8}$$

in order for an equilibrium with respect to investment levels k_i and prices p_i to be determined *simultaneously*[9].

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[8] Which does not depend on population density (*d*).

[9] It is relatively easy to check that second order conditions for maximum are satisfied.

Setting $k_i = k$ and $p_i = p$, we can obtain the symmetric solution, which reduces to the solution of the following system:

$$p_i = \frac{t}{n \cdot k}, \ (9) \qquad \text{and} \qquad k_i = \frac{d \cdot p}{2n}, \ (10)$$

whose solution gives[10]:

$$p^* = \sqrt{\frac{2t}{d}}, \ (11) \qquad \text{and} \qquad k^* = \frac{1}{n} \cdot \sqrt{\frac{d \cdot t}{2}}. \ (12)$$

The expressions for p^* and k^* in (11) and (12) can be substituted into the profit function in (6), in order for the individual short-run profit to be determined:

$$\Pi_i^* = \frac{1}{n} \cdot \sqrt{\frac{d \cdot t}{2}}. \tag{13}$$

Observe that, following (13), the zero-profit condition requires that infinitely many firms enter into the market. The long-run (free-entry) equilibrium is only achieved when infinitely many firms enter the market, investing $k = 0$ each, in order to monopolise an infinitely small part of the market. Therefore, as long as fixed costs are treated as a strategic variable of potential entrants, which is directly related with the market potential that each one of them covers in equilibrium (note from (12) the inverse relation between k^* and *n*) infinitely many firms will enter the market. In that case, each firm's market area is zero which also determines that, in the long run, no investment in transportation-reducing infrastructure (or, in terms of Grossman and Shapiro, 1984, informative advertising) will take place.

The fact that unit transportation costs are infinite for zero individual investment is responsible for the result according to which the infinitely large number of firms results in a monopoly-like situation rather than in a more competitive market. A higher number of firms results in less investment in transportation infrastructure, which implies more market power for each oligopolist. Furthermore, short-run individual profit, which in this particular model equals individual investment, is a decreasing function of the number of firms. Therefore, the higher the number of firms, the stronger the incentives for individual firms to make their products less substitutable with the varieties offered by their rivals.

Substituting $\tau = \frac{t}{k^*}$, where $k^* = \frac{1}{n} \cdot \sqrt{\frac{d \cdot t}{2}}$, in the social cost function and assuming that the policy maker considers that transportation and entry costs should have different weights on social costs, by using a parameter $\lambda \in (0,1)$, the function to be minimised is:

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[10] These results have also been derived in a simpler version in Barreda *et al.* (2000).

$$SC(\lambda) = \lambda \cdot k^* \cdot n + (1-\lambda) \cdot 2 \cdot d \cdot n \cdot \int_0^{\frac{1}{2n}} \tau \cdot x dx \tag{14}$$

We get that social cost does not depend on the number of firms (*n*):

$$SC(\lambda) = \frac{(\lambda + 1)\sqrt{dt}}{\sqrt{2^3}}. \tag{15}$$

Hence, in this model, *state intervention by setting an optimal number of firms is not an effective policy*. Social costs will be higher, the higher the importance of entry costs in the social cost function, the higher the population density, and the higher the infrastructure deficiencies reflected in *t*.

Following this rather extreme result, we will consider any exogenous number of firms $\overline{n}$ in order for the optimal public policy to be determined with respect to public investment in infrastructure (*I*).

We minimise the following social cost function, which assigns different weights to entry costs and transportation costs:

$$SC(\lambda) = EC + TC = \lambda \cdot \left(I + k^* \cdot \overline{n}\right) + (1-\lambda) \cdot 2 \cdot d \cdot \overline{n} \cdot \int_0^{\frac{1}{2n}} \tau \cdot x dx, \tag{16}$$

where we have substituted *t* by $\frac{w}{I}$ so that $k^* = \frac{1}{\overline{n}} \cdot \sqrt{\frac{d \cdot w}{2I}}$ and $\tau = \frac{w}{k^* \cdot I}$.

The expression we obtain is:

$$SC(\lambda) = \frac{4\lambda I\sqrt{(dwI)} + \lambda dw\sqrt{2} + dw\sqrt{2}}{4\sqrt{(dwI)}}. \tag{17}$$

Minimising (17) with respect to public investment we get that the optimal public investment is[11]:

$$I^o = \frac{1}{4\left(\sqrt[3]{\lambda}\right)^2} \sqrt[3]{2 \cdot d \cdot w}\left(\sqrt[3]{\lambda + 1}\right)^2. \tag{18}$$

The optimal public investment depends positively on the population density (*d*) and on the toughness of natural conditions (*w*). It will coincide with the result of the non-weighted minimisation when $\lambda = \frac{1}{2}$, it will increase as the relative importance of entry costs λ decreases to *0*, until it attains an infinite value, and it will decrease to a minimum of $\frac{1}{2}\sqrt[3]{d \cdot w}$, as λ grows to 1.

[11] It can be checked that the S.O.C. for minimum holds.

Substituting the optimal public investment in the social cost function we get the optimal social cost:

$$SC(\lambda)^o = \frac{3}{4}\sqrt[3]{2\lambda dw}\left(\sqrt[3]{(\lambda+1)}\right)^2. \tag{19}$$

This function depends positively on the relative importance of entry costs λ, population density (d), and geographical difficulties (w)[12].

Similarly, substituting the optimal public investment in the equilibrium individual price, investment, and profit of each firm we get that their values are[13]:

$$p^o = \sqrt{\frac{2w}{dI^o}} = \frac{2}{d}\sqrt{\left(\frac{\left(\sqrt[3]{2\lambda dw}\right)^2}{\left(\sqrt[3]{(\lambda+1)}\right)^2}\right)}. \tag{20}$$

$$k^o = \Pi^o = \frac{1}{n}\sqrt{\frac{dw}{2I^o}} = \frac{1}{n}\sqrt{\left(\frac{\left(\sqrt[3]{2\lambda dw}\right)^2}{\left(\sqrt[3]{(\lambda+1)}\right)^2}\right)}. \tag{21}$$

Proposition 1: *In the case of endogenous entry costs (k_i) and optimal public investment I^o, equilibrium prices depend positively on the relative importance of entry costs λ, and the toughness of natural conditions (w), and negatively on population density (d). The equilibrium individual investment and profit, which are equal, depend positively on λ, w, and d, and negatively on the number of firms (n).*

Proof: See Appendix.

If both the central authority and the firms invest in transportation cost reducing infrastructure, private and public investment increase with the population density and the toughness of natural conditions, but public investment decreases with increases in the relative weight of entry costs, while private investment increases with them in order to compensate the decrease in public investment[14], and

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[12] $$\frac{\partial SC(\lambda)^o}{\partial \lambda} = \frac{\sqrt[3]{2dw}(3\lambda+1)}{4\sqrt[3]{(\lambda+1)}\left(\sqrt[3]{\lambda}\right)^2} > 0.$$

[13] These results have also been derived in Barreda *et al.* (2002).

[14] It is interesting to observe that, despite the positive interaction of private and public investment in reducing unit transportation costs (as their product appears in the denominator of the unit transportation cost), in strategic terms, the two magnitudes are substitutable. Consider the derivative: $\frac{\partial k^o}{\partial I^o} = -\frac{\sqrt{2dw}}{4n\left(\sqrt{I^o}\right)^3}$. It can be easily seen that the (negative) effect of public investment on private investment is greater the greater the population density, the greater the natural difficulties, the lower the number of firms, and the lower the level of public investment.

decreases with the number of firms in the market, because all of them share equally the burden of private investment. Social costs increase with increases in population density, natural difficulties and in the relative importance of entry costs. The firms' prices increase with the relative importance of entry costs and with an increase in the natural difficulties, while they decrease with an increase in population density.

4. Optimal Public Investment generalised to n Regions with a binding budget constraint

Let us study the case in which the budget is less than the unconstrained optimal public investment $\left(B < I^o\right)$. We are going to consider that there are n regions with different geographic difficulties (w_1, w_2,...,w_n) and different population densities (d_1, d_2,...,d_n), in which a central authority has to decide how much to invest in transportation infrastructure (I_1, I_2,...,I_n), apart from the quantity privately invested by firms (k_i).

Following (17) total social cost will be:

$$TSC = SC_1 + SC_2 + \ldots + SC_n = \frac{4\lambda I_1\sqrt{(d_1 w_1 I_1)} + \lambda d_1 w_1\sqrt{2} + d_1 w_1\sqrt{2}}{4\sqrt{(d_1 w_1 I_1)}} +$$

$$+\frac{4\lambda I_2\sqrt{(d_2 w_2 I_2)} + \lambda d_2 w_2\sqrt{2} + d_2 w_2\sqrt{2}}{4\sqrt{(d_2 w_2 I_2)}} + \ldots + \frac{4\lambda I_n\sqrt{(d_n w_n I_n)} + \lambda d_n w_n\sqrt{2} + d_n w_n\sqrt{2}}{4\sqrt{(d_n w_n I_n)}} \quad (22)$$

Let us consider the following budget constraint: $I_1 + I_2 + \ldots + I_n = B$. In order to obtain the optimal public investment we set up the Lagrangian using (22):

$$L = TSC - \mu\left(I_1 + I_2 + \ldots + I_n - B\right). \quad (23)$$

From the F.O.C. $\frac{\partial L}{\partial I_1} = 0$, $\frac{\partial L}{\partial I_2} = 0$,..., $\frac{\partial L}{\partial I_n} = 0$ and $\frac{\partial L}{\partial \mu} = 0$, we obtain the candidates for the constrained optimal investment:

$$I_1^{'} = \frac{1}{4\left(\sqrt[3]{(\lambda - \mu)}\right)^2}\sqrt[3]{2 \cdot d_1 \cdot w_1}\left(\sqrt[3]{(\lambda + 1)}\right)^2, \quad (24)$$

$$I_2^{'} = \frac{1}{4\left(\sqrt[3]{(\lambda - \mu)}\right)^2}\sqrt[3]{2 \cdot d_2 \cdot w_2}\left(\sqrt[3]{(\lambda + 1)}\right)^2. \quad (25)$$

$$\ldots\ I_n^{'} = \frac{1}{4\left(\sqrt[3]{(\lambda-\mu)}\right)^2}\sqrt[3]{2\cdot d_n\cdot w_n}\left(\sqrt[3]{(\lambda+1)}\right)^2. \tag{26}$$

Using the budget constraint equation, we can obtain the optimal public investment in each of the n regions[15]:

$$I_1^o = \frac{\sqrt[3]{d_1\cdot w_1}}{\sqrt[3]{d_1\cdot w_1}+\sqrt[3]{d_2\cdot w_2}+\ldots+\sqrt[3]{d_n\cdot w_n}}B, \tag{27}$$

$$I_2^o = \frac{\sqrt[3]{d_2\cdot w_2}}{\sqrt[3]{d_1\cdot w_1}+\sqrt[3]{d_2\cdot w_2}+\ldots+\sqrt[3]{d_n\cdot w_n}}B. \tag{28}$$

$$\ldots\ I_n^o = \frac{\sqrt[3]{d_n\cdot w_n}}{\sqrt[3]{d_1\cdot w_1}+\sqrt[3]{d_2\cdot w_2}+\ldots+\sqrt[3]{d_n\cdot w_n}}B. \tag{29}$$

Observe that the optimal investments are expressed as percentage shares of the budget, and they do not depend on λ. The critical magnitudes in order to decide the sharing of the budget will be the product between density and natural difficulties in each region. It could even be the case that a region with low natural difficulties and high population density should receive a lower share of the budget than a region with low population density and very high natural difficulties.

In this case, the investment of public money in one or the other region depends on the differential characteristics between them, regarding population densities and natural difficulties, and not on the relative importance of entry costs, which we assume equal in both regions, or on the number of firms in each region, which, as we already know, is ineffective in order to affect social welfare.

Proposition 2: *Optimal public investment in infrastructure in the case of endogenous entry costs and n different regions, depends positively on own region density, own natural difficulties, and the budget, and negatively on the other regions ones.*

Proof: See Appendix.

Regarding the firm's decisions in each region, we will report here the socially optimal individual prices, investment and profits of each firm in Region n:

$$p_n^o = \sqrt{\frac{2w_n}{d_n I_n^o}} = \sqrt{\frac{2\left(\sqrt[3]{(d_n w_n)}\right)^2\left(\sqrt[3]{(d_1 w_1)}+\sqrt[3]{(d_2 w_2)}+\ldots+\sqrt[3]{(d_n w_n)}\right)}{Bd_n^2}}. \tag{30}$$

[15] It can be checked that the S.O.C for minimum hold.

Proposition 3: *The price of each firm in Region n when the central authority's investment in infrastructure is socially optimal depends positively on natural difficulties, and other regions densities, and negatively on own density and budget.*

Proof: See Appendix.

The price charged by firms in Region n will increase as long as the natural difficulties, in itself or in the other regions, increase, and with increases in the other regions' populations, and it will decrease with increases in its own population density or in the available budget. This may suggest an explanation of why prices in a poor country may be higher than in one with a less stringent budget constraint. The suboptimal public provision of transportation infrastructure allows for higher levels of unit transportation cost, hence, for a lower degree of competition, so the firms are able to charge higher prices.

$$k_n^o = \Pi_{in}^o = \frac{1}{n_n}\sqrt{\frac{d_n w_n}{2I_n^o}} = \frac{1}{n_n}\sqrt{\frac{\left(\sqrt[3]{(d_n w_n)}\right)^2\left(\sqrt[3]{(d_1 w_1)}+\sqrt[3]{(d_2 w_2)}+\ldots+\sqrt[3]{(d_n w_n)}\right)}{2B}}\,. \qquad (40)$$

Proposition 4: *The individual investment and profit of each firm when the central authority's investment in infrastructure is optimal in Region n depends positively on natural difficulties and population densities and negatively on the number of firms in the region and the Budget.*

Proof: It is straightforward, and we omit it.

Individual investment and profit for a firm in a given region depend positively on increases in any of the regions population densities and natural difficulties, and negatively on an increase in the budget, or an increase in the number of firms located in that region.

For a given number of firms, profits will be higher in the region with higher population density and natural difficulties. Also, with perfect capital mobility across regions, we would expect higher entry in the region with higher population and worse natural conditions until profits are equalised.

5. Empirical Results

In Appendix 7.5 we present the data on road kilometres built, square kilometres, number of inhabitants, number of firms, population density, number of parliament seats, state of the transportation infrastructure[16], and budgeted public investment by the central Spanish government in each region on year 2004.

These data were used to construct Table 1 below, where we can find the observed sharing of the budget among the regions and compare it to the optimal (social cost minimizing) sharing calculated according to our theoretical result of the previous section (Equation 29).

Region	Real Public Investment (1)	Observed % (2)=(1)/Total	(d*w)^(1/3) (3)	Optimal % (4)=(3)/Total	Difference (5)=(2)-(4)
Andalucía	405.965,49	17,6%	6,79	5,4%	12,18%
Castilla y León	328.016,69	14,2%	5,09	4,1%	10,16%
Aragón	232.159,59	10,1%	4,82	3,8%	6,22%
Extremadura	211.750,22	9,2%	4,94	3,9%	5,24%
Castilla-La Mancha	158.788,85	6,9%	3,85	3,1%	3,81%
Asturias	194.064,89	8,4%	5,99	4,8%	3,64%
Galicia	179.968,78	7,8%	5,41	4,3%	3,49%
Cantabria	171.191,97	7,4%	5,99	4,8%	2,65%
Comunidad Valenciana	145.849,45	6,3%	8,12	6,5%	-0,14%
Cataluña	132.313,92	5,7%	8,24	6,6%	-0,83%
Rioja	23.197,17	1,0%	5,39	4,3%	-3,29%
Canarias	57.289,48	2,5%	7,67	6,1%	-3,62%
Navarra	101,26	0,0%	5,32	4,2%	-4,23%
Murcia	16.642,76	0,7%	7,02	5,6%	-4,86%
Balears	8.181,82	0,4%	7,63	6,1%	-5,72%
País Vasco	0,00	0,0%	7,92	6,3%	-6,31%
Madrid	31.826,21	1,4%	11,95	9,5%	-8,14%
Ceuta y Melilla	10.810,59	0,5%	13,50	10,7%	-10,28%
TOTAL	**2.308.119,14**	**1,00**	**125,63**	**1**	

Source: INE and Presupuestos Generales del Estado (2006)

Table 1: Observed vs. Optimal sharing of the Budget.

It is easy to observe from the table above that the actual sharing of the budget does not coincide with our theoretically optimal sharing except for some regions: Valencian Community and Catalonia. For the other regions there are differences of up to plus/minus 10%.

We are going to check now if at least the population density and the bad state of the existing infrastructure play the positive role they should have in the sharing of the budget according to our

[16] We have built this proxy of the natural difficulties or bad state of the existing infrastructure (*w*) by dividing the number of square kilometres of the region by the number of road kilometers.

theoretical results. We have carried out a linear regression analysis[17] of the observed sharing of the budget on the density (d) and the state of the existing infrastructure (w).

R^2	Corrected R^2	F	Reg. DF	Res. DF	Sig.
0,397	0,311	4,609	2	14	0,029
	Beta	**S.D.**	**Standard B**	**t**	**Sig.**
Constant	-0,004	0,04		-0,099	0,922
Density	-0,00000682	0,00	-0,222	-0,983	0,342
W	0,026	0,012	0,507	2,242	0,042**

Table 2: Regression Analysis of the Observed Investment Budget Share on w and density.

Only the bad state of the existing infrastructure has the positive expected sign and has a significant impact on the sharing of the budget. Surprisingly the population density does not significantly contribute to explain the central government investment.

Faced to this puzzle we have looked for a possible omitted relevant variable which could be influencing the public investment decision instead of the population density. We have turned to the electoral data on number of parliament seats corresponding to each region and we have repeated the regression.

R^2	Corrected R^2	F	Reg. DF	Res. DF	Sig.
0,542	0,472	8,28	2	14	0,004
	Beta	**S.D.**	**Standard B**	**t**	**Sig.**
Constant	-100828,607	65518,067		-1,539	0,146
Seats	3230,102	1353,060	0,440	2,387	0,032**
W	60824,025	21861,842	0,513	2,782	0,015**

Table 2: Regression Analysis of the Observed Investment Budget Share on w and seats.

In this regression both the bad state of the existing infrastructure and the number of parliament seats to be obtained from the region have a significant impact of the expected sign in the sharing of the budget. The number of parliament seats has some relationship to population density but it is not purely proportional, so some political considerations could be getting some attention in this kind of investment decisions apart from the economical considerations.

6. Conclusions

[17] For the empirical analysis we did not use the data on Ceuta and Melilla, given that their small surface

In this paper, we have modified the monopolistic competition framework in order to account for two facts: first, the relative weights of transportation and entry costs in the social cost function may play an important role in the relationship between the free entry and the optimal number of firms in the market; Second, the consideration of exogenous or endogenous entry costs may be crucial for the effectiveness of public intervention by regulating entry into the market.

Then, we apply the model to the case of exogenous entry costs and we show that, depending on the relative weight of transportation costs and entry costs in the social loss function, the optimal results in terms of number of firms, investment, etc. can be significantly away from those provided by the implicit assumption taken in the literature that both have equal weights. Therefore, when entry costs are exogenous, the socially optimal number of firms may coincide with the free entry equilibrium provided that entry costs are relatively not very important as compared to the transportation costs.

In the case of endogenous entry costs, setting the number of firms by a central planner is not an effective policy. Instead, public investment in a transportation cost-reducing infrastructure will be an effective policy, and its level will depend on the relative weights of entry and transportation costs in the social cost function. Firms' entry costs and firm-specific unit transportation costs are endogenously determined. The result on long run (free entry) equilibrium coincides then with the social optimum but it requires that an infinity of firms enter into the market. This result depends on the fact that (sunk) entry costs are totally endogenous. We are conscious of the fact that with some fixed (exogenous) part which is a necessary minimum to be paid by firms entering into the market, the free-entry equilibrium number of firms would be bounded from above by a number which increases to infinity as the exogenous part of sunk costs decreases to zero. However, we have used the extreme case of totally endogenous entry costs to illustrate a case in which entry control, which is more common in the literature than in real-world policy-making, may be unnecessary. In fact, we have shown that public investment in transportation infrastructure leads to a social optimum which is independent of the number of firms.

In the case of endogenous entry costs and two different regions, the natural difficulties and the population density will play a positive role in the amount of investment in the region by the central authority and also in the number of firms which will enter its market. Besides, in the case of a binding budget constraint the weights of entry and transportation costs will not play a role in the sharing of the budget. Contrary to standard intuition, individual equilibrium prices and profits will tend to be higher in poorer countries.

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and big population density constitute a rather special case in Spain.

In a second phase of the study we have confronted the theoretical results of our framework to real statistical data for 18 Spanish regions. We obtain that a worse existing infrastructure has the positive expected influence in the sharing of the budget but the population density does not have a significant influence. Surprisingly, or maybe not, the number of parliamentary seats to be obtained from a given region has a significant positive impact on the sharing of the budget.

7. Appendix

7.1 Restrictions on the Reservation Price

Following the results obtained in the paper, the assumption that consumers' reservation price must be higher than the generalised price for any of them $\left(R > p + \tau \cdot \frac{1}{2n} \right)$ in order that we have full market coverage, implies for the model with endogenous entry costs: $R > \frac{3}{2}\sqrt{\frac{2w}{dI}}$. In the social optimum, that will be:

$$R > 3\sqrt{\left(\frac{\left(\sqrt[3]{2\lambda w}\right)^2}{\left(\sqrt[3]{d^2(\lambda+1)}\right)^2} \right)}.$$

7.2 Proof of Proposition 1

$$\frac{\partial p^o}{\partial \lambda} = \frac{2}{3}\left(\sqrt[3]{2}\right)^2 \frac{w}{\sqrt{\left(\left(\sqrt[3]{2}\right)^2 \frac{\left(\sqrt[3]{\lambda d w}\right)^2}{\left(\sqrt[3]{(\lambda+1)}^2\right)} \right)} \sqrt[3]{(\lambda d w)} \left(\sqrt[3]{(\lambda+1)}\right)^5} > 0.$$

$$\frac{\partial p^o}{\partial d} = -\frac{4\sqrt[3]{2}}{3d\sqrt{\left(\frac{\left(\sqrt[3]{\lambda d w}\right)^2}{\left(\sqrt[3]{(\lambda+1)}^2\right)} \right)} \sqrt[3]{(\lambda d w)} \left(\sqrt[3]{(\lambda+1)}\right)^2} \lambda w < 0.$$

$$\frac{\partial \Pi^o}{\partial \lambda} = \frac{1}{3}\left(\sqrt[3]{2}\right)^2 y \frac{w}{n\sqrt{\left(\left(\sqrt[3]{2}\right)^2 \frac{\left(\sqrt[3]{\lambda d w}\right)^2}{\left(\sqrt[3]{(\lambda+1)}^2\right)} \right)} \sqrt[3]{(\lambda d w)} \left(\sqrt[3]{(\lambda+1)}\right)^5} > 0. \quad \textbf{QED}$$

7.3 Proof of Proposition 2

$$\frac{\partial I_n^o}{\partial d_n} = \frac{1}{3} B \sqrt[3]{w_n} \left(\sqrt[3]{d_1} + \sqrt[3]{d_2} + \ldots + \sqrt[3]{d_{n-1}} \right) \frac{\sqrt[3]{w_1} + \sqrt[3]{w_2} + \ldots + \sqrt[3]{w_{n-1}}}{\left(\sqrt[3]{d_n} \right)^2 \left(\sqrt[3]{d_1}\sqrt[3]{w_1} + \sqrt[3]{d_2}\sqrt[3]{w_2} + \ldots + \sqrt[3]{d_n}\sqrt[3]{w_n} \right)^2} > 0.$$

$$\frac{\partial I_1^o}{\partial d_n} = -\frac{1}{3} B \sqrt[3]{w_n} \sqrt[3]{d_1} \frac{\sqrt[3]{w_1}}{\left(\sqrt[3]{d_n} \right)^2 \left(\sqrt[3]{d_1}\sqrt[3]{w_1} + \sqrt[3]{d_2}\sqrt[3]{w_2} + \ldots + \sqrt[3]{d_n}\sqrt[3]{w_n} \right)^2} < 0. \textbf{QED}$$

7.4 Proof of Proposition 3

$$\frac{\partial p_n^o}{\partial d_n} = -\frac{1}{6} \frac{\sqrt{2}}{\left(\sqrt[3]{d_n} \right)^5} \sqrt[3]{w_n} \frac{4\left(\sqrt[3]{d_1}\sqrt[3]{w_1} + \sqrt[3]{d_2}\sqrt[3]{w_2} + \ldots + \sqrt[3]{d_{n-1}}\sqrt[3]{w_{n-1}} \right) + 3\sqrt[3]{d_n}\sqrt[3]{w_n}}{\sqrt{\left(\sqrt[3]{d_1}\sqrt[3]{w_1} + \sqrt[3]{d_2}\sqrt[3]{w_2} + \ldots + \sqrt[3]{d_n}\sqrt[3]{w_n} \right)}\sqrt{B}} < 0. \textbf{QED}$$

7.5 Data on the spanish regions

The last available data on road kilometers built in each region when elaborating this article referred to 2004, so all other quantities also relate to this year: number of inhabitants, number of parliament seats, number of firms, and public investment budgeted for 2005. One limitation of the data is that the regional government investments are not included, only the central government ones are considered, as we were interested in studying the sharing of the central government infrastructure investment budget.

Region	**Road Km (1)**	**Inhabitants (2)**	**Square Km (3)**	**Density (2)/(3)**	**W (3)/(1)**	**Parliament Seats**	**Nº Firms**	**Public Investment (Thousands €)**
Andalucía	24.558	7.687.518	87.590	87,77	3,57	61	441.623	405.965,49
Castilla y León	18.890	2.493.918	93.814	26,58	4,97	33	155.004	328.016,69
Aragón	11.170	1.249.584	47.698	26,20	4,27	13	85.814	232.159,59
Extremadura	8.923	1.075.286	41.634	25,83	4,67	10	55.568	211.750,22
Castilla-La Mancha	32.319	1.848.881	79.409	23,28	2,46	20	113.967	158.788,85
Asturias	5.001	1.073.761	10.604	101,26	2,12	8	67.039	194.064,89
Galicia	17.411	2.750.985	29.574	93,02	1,70	24	180.977	179.968,78
Cantabria	2.586	554.784	5.253	105,61	2,03	5	35.649	171.191,97
Comunidad Valenciana	8.498	4.543.304	23.254	195,38	2,74	32	315.214	145.849,45

Cataluña	12.176	6.813.319	32.091	212,31	2,64	47	543.719	132.313,92
Rioja	1.872	293.553	5.028	58,38	2,69	4	21.049	23.197,17
Canarias	4.247	1.915.540	7.447	257,22	1,75	15	120.294	57.289,48
Navarra	3.891	584.734	9.801	59,66	2,52	5	39.679	101,26
Murcia	3.748	1.294.694	11.313	114,44	3,02	9	82.484	16.642,76
Balears	2.154	955.045	4.992	191,32	2,32	8	85.425	8.181,82
País Vasco	4.250	2.115.279	7.089	298,39	1,67	19	155.666	0,00
Madrid	3.400	5.804.829	8.022	723,61	2,36	35	436.074	31.826,21
Ceuta y Melilla	58	142.670	32	4458,44	0,55	2	7.338	10.810,59
TOTAL	**165.152**	**43.197.684**	**504.645**	**85,60**	**3,06**	**350**	**2.942.583**	**2.308.119,14**

Source: INE and Presupuestos Generales del Estado (2006)

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