Testing Value vs Waiting Value: a more general approach to Environmental Decisions under Uncertainty and Irreversibility

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This Version: October 2006

Abstract The paper analyzes environmental decisions under uncertainty and irreversibility, by introducing a value - the $Testing\ Value$. This value emerges in all those situations in which the level of information concerning future economic benefits of development (and its future environmental costs) depends on the level of development carried out. We show that including the Testing Value into the analysis could push the decision maker towards a higher level of preservation of the environmental resource. The reason is that the Testing Value leads the decision maker to develop only a certain amount of the environmental asset (*internal solutions*); on the contrary, the Waiting or Quasi-Option Value (widely used in the environmental literature) would lead more frequently to *corner solutions*. In other words, we prove that when the decision maker is faced with an intertemporal environmental choice problem where information is endogenous, destroying now a small part of an environmental resource induces her to destroy less in the future.

JEL references: C61; D81; Q32.

Key Words: Testing value; Uncertainty; Irreversibility.

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1 Introduction

1.1 The "history" of the Waiting Value

The issue of irreversibility and uncertainty in environmental decisions has been largely analyzed in the last three decades. From the first definition of the *Quasi-Option Value* (*QOV*), given by Arrow and Fisher (1974), the key concept has been developed in several articles, among others, by Henry (1974), Dasgupta and Heal (1979), Hanemann (1982, 1989), Fisher and Krutilla (1985), Beltratti, Chichilnisky and Heal (1996), Fisher (2000), Pindyck (2000).

Arrow and Fisher (1974), while examining the optimal level of development of a natural resource, identified a concept that they termed "quasi-option" value. The concept emerged from a two-period model of choice (develop or preserve), where development is irreversible [3] and the expected net benefits of preservation in future periods are conditional upon the choice in the present period.

Fisher and Hanemann (1987) [4] started their analysis using the same context of Arrow and Fisher's model, characterized by *risk neutrality* of the *D*ecision *M*aker (*DM* henceforth), *irreversibility* of the action "development", *uncertainty* about the future benefits (of development and preservation) and by independent learning (*exogenous information*): the *DM* can receive *new* information about the environmental asset (about the future benefits of his action) only by letting time pass; the acquisition of information is independent from the choice made in the first period.[5] More specifically, in Fisher and Hanemann's model, there are two alternative scenarios for the acquisition of information about the future consequences of development. In the first scenario, *exogenous* information is *available* (and it is known it will be available) *with certainty* at the end of the first period, in sufficient time to be incorporated into the decision to be taken in the second (and last) period; in this scenario, the prospect of future information is fully recognized and incorporated explicitly in the current decision. In the second information scenario, either information is *not available* (and it is known that it will be not available) in sufficient time to be incorporated into the choice in the future period or it is *disregarded* by the *DM* when he sets the current level of

[3] In the sense that development of the environmental resource can take place either "now" or "in the future" but, once undertaken, it is irreversible.

[4] See also Hanemann (1989), Fisher (2000).

[5] In particular, information can emerge by only "waiting" (as the second period approaches, one is able to make a more accurate assessment of the social value of wilderness preservation in that period) or as the result of a separate research program.

development.

Let us identify with c_t the amount of environmental resource preserved in period $t = 1, 2$, with $q \in [0, 1]$ the probability of acquiring new information exogenously at the end of period 1 and assume that the level of the environmental resource is normalized to 1. If we define with $EV_{exo|q=1}(c_1)$ the expected net benefits over both periods (of preserving c_1 in the first period) under the first information scenario [6] and with $EV_{no}(c_1)$ the same expected net benefits under the second information scenario, requiring the investment decision to be confined to a binary choice between full development ($c_1 = 0$) and no development at all ($c_1 = 1$), Fisher and Hanemann define as option value *à la* Arrow-Fisher (*Quasi-Option Value*)

$$QOV = \left[EV_{exo|q=1}(1) - EV_{exo|q=1}(0)\right] - \left[EV_{no}(1) - EV_{no}(0)\right]$$

This is a correction factor, that can be interpreted in the following way: let us rewrite the QOV as

$$QOV = \left[EV_{exo|q=1}(1) - EV_{no}(1)\right] - \left[EV_{exo|q=1}(0) - EV_{no}(0)\right]$$

In the terminology of decision theory, $\left[EV_{exo|q=1}(1) - EV_{no}(1)\right]$ is the value of perfect information conditional on having preserved the whole environmental area in $t = 1$ (it is the gain deriving from exogenous information when choosing c_2 conditional on having set $c_1 = 1$). Similarly, $\left[EV_{exo|q=1}(0) - EV_{no}(0)\right]$ is the value of perfect information conditional on having destroyed the whole environmental area in $t = 1$. The QOV is the difference between these two values. But irreversibility creates asymmetry: if one decides to preserve initially, he can always reverse that decision later when he obtains more accurate information about the consequences of development; on the contrary, if he decides to develop (everything) now ($c_1 = 0$), the decision cannot be reversed and any subsequent information he may receive has no economic value. Hence, $EV_{exo|q=1}(0) = EV_{no}(0)$ and the expression of the option value *à la* Arrow-Fisher becomes

$$QOV = EV_{exo|q=1}(1) - EV_{no}(1)$$

Consequently, the QOV is always non-negative,[7] since (exogenous) information is not dangerous, so that $EV_{exo|q=1}(1) - EV_{no}(1) > 0$. A decision to set

[6] Remember that in the first information scenario new information comes about *exo*genously with certainty, hence $q = 1$.

[7] However, when there is a continuum of preservation (development) levels, rather than a binary choice between full development and full conservation, this conclusion needs to be modified. Let $c_1 \in [0, 1]$ and $c_2 \in [0, c_1]$, because of irreversibility. Analyzing this case, Epstein (1980) established that it is not necessarily true that $(c_1^*)_{exo|q=1} \geq (c_1^*)_0$. He developed also a set of sufficients conditions for Arrow and Fisher's result to carry over when there is a continuum of preservation (development) levels.

$c_1 = 1$ preserves flexibility, and the QOV is the value of such flexibility. In particular, it is the gain the DM obtains when he can receive (exogenous) information regarding future benefits, *if* he decides not to develop in the current period (with respect to the case in which he ignores the possibility of receiving this kind of information).[8]

This does not mean that developing in the first period should never be optimal, hence $(c_1^*)_{exo|q=1}$ could be different from 1; after all, it may happen that $EV_{exo|q=1}(1) < EV_{exo}(0)$.

Rather, it means that the case for preservation is strenghtened when one recognizes the prospect of further information about the future consequences of development: in general, the amount of environmental resource preserved under the first information scenario is higher, i.e. $(c_1^*)_{exo|q=1} \geq (c_1^*)_{no}$.

According to the definition of Arrow and Fisher (1974), the QOV is a particular "Waiting Value", i.e. a value emerging when the DM "stands by" in the current periods, moving his decision to the future when new (exogenous) information may be available. Differently from the definition above, the interpretation we give of the waiting value (intended as the money the DM is willing to pay in order to shift the decision from now to the future) is closer to the formulation of Conrad (1980) and Miller and Lad (1984): Conrad (1980) states that the QOV - so defined - is identical to the expected net value at time $t = 1$ of information in c_1; Miller and Lad (1984) state that "the existence of a QOV, as defined generally, depends only the name on the irreversible character of development. Under conditions of irreversibility, an option value is called a QOV. But any option value, quasi or otherwise, stems from the relative values of flexible and inflexible decisions, not from the existence of irreversibility per se".

In line with Conrad's and Miller's and Lad's intuitions, we agree that, in general, a waiting value (whose family the QOV belongs) has to be interpreted as the difference between the expected value of the *optimal flexible decision* and that of the *optimal fixed decision*: this difference is greater than zero. Hence, we define in general the *Waiting Value* (WV henceforth) as

$$WV = EV_{exo}((c_1^*)_{exo}) - EV_{no}((c_1^*)_{no})$$

In the first information scenario, the DM can choose c_2 after having received (with a given probability $q \in (0, 1]$) information about the relative

[8] Another interpretation: if the DM ignored the possibility of exogenously acquiring information about the future benefits of development/preservation and myopically based his decision on the maximization of $EV_{no}(c_1)$, QOV is the shadow tax that would have to be imposed on development in order to steer him the correct choice on whether or not to develop at all.

benefits of the second period (what Miller and Lad called *flexible decision*); hence, we indicate with $EV_{exo}(c_1)$ the expected value of net benefits over both periods (of preserving c_1 in the first period) when (only) exogenous information arrives with probability $q \in (0, 1]$ at the end of period 1.

In the second information scenario, where the myopia of the DM prevents him from recognizing the possibility of acquiring new information exogenously, the optimal choices of c_1 and c_2 are made simultaneously in the first period (what Miller and Lad called *inflexible decision*). [9]

According to our formulation of the problem, the QOV *à la* Arrow-Fisher becomes a particular case of WV that comes around when the DM chooses to preserve the entire environmental resource in the first period: it is the expected value of information conditional on having set $c_1^* = 1$. Also, it has to be intended as the upper bound of the WV, since, by preserving everything in the first period, the DM, in the second period, can choose an action in the same set of possible actions available in the first period. Hence, $QOV \geq WV$.[10]

1.2 The need for a Testing Value

The conclusions drawn by Arrow and Fisher (1974) and Fisher and Hanemann (1987) on the QOV still hold, even if the cost of information is included in the model.[11]

On the other hand, independently from the fact that information arrives at a cost or not, Arrow and Fisher's results on the optimality of a complete preservation of environmental resources when their destruction is irreversible are derived in the framework of *independent learning*, i.e. with exogenous information.

It is commonly accepted in the literature on environmental option values that this result does not hold if *information* is *endogenous* (i.e., *dependent*

[9] Hence the DM chooses the amount of c_2 without knowing the realizations of the second period benefits.

[10] Fisher and Hanemann (1987) suppose that, in what we called the "exogenous information scenario", information arrives with probability $q = 1$. We instead generalise such framework by allowing information to come out not with certainty, but according to a given probability $q \in (0, 1]$. See Section 1.3.2 for a complete analysis of this more general case.

[11] They recognize the fact that information about the consequences of development arriving automatically, by simply letting time get by, is unrealistic (or, at least, rare): the acquisition of information usually requires the expenditure of resources and occurs only if some (other) agents take appropriate actions. Indeed, if the cost of information exceeded the expected value of information given $c_1 = 1$, the difference $EV_{exo|q=1}(1) - EV_{no}(1)$ would be negative, hence it would not be optimal for the DM to acquire it.

learning). Miller and Lad (1984) and Freeman (1984) stated that "*if information concerning future effects of the irreversible depletion of an environmental resource can be obtained only carrying out depletion itself in $t = 1$, than it is optimal to develop (only) one portion of the environmental asset in the current period*". In other words, the policy of postponing the choice in order to enable the DM to profit from the coming information is sub-optimal when this is endogenous: if the uncertainty is primarily about the benefits of preservation/development, this strengthens the case for some development. On the other hand, even when more information is provided solely by development, *substantial* development [12] may not be in order.

Freeman (1984) and Fisher and Hanemann (1987) assume that *full* information is provided by *any* amount of development; moreover, no exogenous information arrives. Let us identify with $\lambda \in [0, 1]$ the probability of acquiring new information endogenously; the resulting expected value function under this scenario, denoted as $EV_{endo|\lambda=1}(c_1)$, is equivalent to that in the "no-information" scenario, EV_{no}, in the event no development is undertaken, and to that in the "new exogenous information" (with certainty) scenario, $EV_{exo|q=1}$, in the event *any* development is undertaken [13]. In symbols, [14]

$$EV_{endo|\lambda=1}(c_1) = \begin{cases} EV_{no}(1) & \text{if } \quad c_1 = 1 \\ EV_{exo|q=1}(c_1) & \text{if } \quad c_1 \in [0, 1) \end{cases}$$

Several results follow form this (particular) formulation of the problem:

1) It can never be optimal to preserve the whole amount of the environmental resource, i.e. $(c_1^*)_{endo} \neq 1$.

2) There is still a corner solution for c_1, in the sense that one either develops fully now, i.e. $(c_1^*)_{endo} = 0$, or engages in an infinitesimal amount $\varepsilon > 0$ of development, i.e. $(c_1^*)_{endo} = \varepsilon$.

3) The *QOV of the minimum feasible development* ($\varepsilon -$ *development*), defined as

$$QOV_\varepsilon = EV_{endo|\lambda=1}(\varepsilon) - EV_{no}(\varepsilon)$$

[12] To be intended as the destruction of a high proportion of the environmental area.

[13] As one can notice, this is a very particular information structure, where only a very particular "kind" of endogenous information is allowed: the new information coming around when any level of development is undertaken is of the same "kind" we discover under the exogenous information context. We maintain this assumption in our model. As in the exogenous information case, we instead do not maintain the assumption that information arrives endogenously with certainty (i.e., $\lambda = 1$) and that the level of the information received is independent from c_1. See Section 1.3.3 for that.

[14] Notice that we indicate with $EV_{endo|\lambda=1}$ the expected value of endogenous information given that it will come with probability $\lambda = 1$ in case of (some) development of the environmental resource.

is always positive.

4) $(c_1^*)_{endo} \geq (c_1^*)_{no}$, i.e. if the decision is to develop when one disregards the possibility of dependent learning, the correct decision when recognizing this possibility cannot be more development and may be less.

The specificity of Freeman's (1984) and Fisher and Hanemann's (1987) results on the QOV_ε raises doubts as to whether the policy implications of the described environmental decision problem could depend on the precise manner in which development generates information (i.e., on the form of the "information production function"). Hence, in our framework (Section 2.1 and 2.2) we try to model endogenous uncertainty in order to be as much "general" and "near to reality" as possible.

First of all, contrarily to what is assumed in the environmental option values literature, we think there are a large number of environmental problems in which the possibility of acquiring new information endogenously depends on the "size" of the development the DM chooses to perform. In the case of oil extraction in one country, for example, there may be uncertainty as to whether and where the land contains oil in commercial quantities. If this is the case, it is likely that the uncertainty can be solved by undertaking some development. But it is doubtless that if you drill the land (by destroying a part of the natural resource) the deeper you drill the higher the probability of discovering an oil well. Another example: if you destroy only one or two trees of the Amazon forest, you obtain very little information about the possible extinction of a certain species. If instead you keep on destroying a larger portion of the forest, you can obtain higher information about the pervasive effects of the development activity. Thus, it seems plausible that, in case information comes out through development of the natural resource, *the level of information coming out must depend on the level of development carried out*. In other words, it must be inversely related to c_1. This is an assumption we introduce in our model, when we characterize the endogenous information framework.

Moreover, differently from Fisher and Hanemann's QOV_ε, we define the *Testing Value* (TV, henceforth) not as the difference between $EV_{endo|\lambda=1}(\varepsilon)$ and $EV_{no}(\varepsilon)$ but rather as

$$TV = EV((c_1^*)_{compl}) - EV_{exo}((c_1^*)_{exo})$$

where $EV(c_1)$ is the expected value of net benefits when there is *both exogenous* (arriving with probability $q \in (0,1]$) *and endogenous* (arriving with probability $\lambda \in (0,1]$, whose level is decreasing in c_1) information[15]; c_1^* is

[15] In what follows, we also define $EV_{|q+\lambda=1}(c_1)$ as the expected value of net benefits

the optimal preservation level in the current period, under this information structure.

According to our definition, the TV has to be interpreted as the *additional value attached to endogenous information*, additional with respect to information exogenously arriving. In other words, it is the gain the DM obtains when he can receive information regarding future benefits, by developing in the current period (with respect to the case in which he ignores the possibility of receiving information in this way). Obviously, if $(c_1^*)_{compl} = 1$, there is only exogenous information, then $EV \equiv EV_{exo}$ and also $(c_1^*)_{exo} = 1$; hence, in this case, $TV = 0$ [16].

The *QOV of the minimum feasible development* defined by Freeman (1984) and Hanemann and Fisher (1987) becomes a particular TV that emerges when the following two conditions are contemporaneously satisfied:

- only endogenous information is available (exogenous information is completely absent);

- information coming out is the same for every $c_1 \in [0, 1)$.

2 The Model

2.1 Assumptions and notation

Let us consider a two-period model of choice ($t = 1, 2$), where in $t = 1$ the DM has to choose the amount of environmental resource he wants to preserve (not develop) until t =2. Assuming the level of the environmental resource is normalized to 1, let us indicate with $c_1 \in [0, 1]$ the amount preserved in period 1. We indicate with b_1 the marginal net benefit deriving from the decision of preserving at time $t = 1$ [17]. We assume current net benefits from preservation are known with certainty by the DM and are negative, i.e. $b_1 < 0$ [18]; thus, the unique incentive to preserve in $t = 1$ is given by the

when both exogenous and endogenous information is available, given that the sum of the probability that it comes out is equal to 1 ($q + \lambda = 1$).

[16] For the properties of the Testing Value, see Section 1.4.3.

[17] Marginal net benefits in period t, b_t, could be interpreted as

$$b_t = (pb_t - pc_t) - (db_t - dc_t)$$

where pb_t, pc_t, db_t, dc_t represent, respectively, marginal preservation benefits, marginal preservation costs, marginal development benefits and marginal development costs in t.

[18] The framework could be further generalized by considering a larger state of nature space, namely

$$\Theta = [b_1(c_1), b_2(c_1, c_2) | (c_1, c_2) \in D]$$

where D is the decision space.

possibility to obtain a positive future benefit in $t = 2$ [19].

In the second period, the DM chooses again the amount of resource to be preserved. Since we assume development is irreversible, it is straightforward that in t =2 it is not possible to preserve more than one has done in t =1: the DM's options in 2 are constrained by the decision made in 1. Thus, if we indicate with c_2 the amount preserved in period 2, by irreversibility it is $c_2 \in [0, c_1]$: the amount chosen in $t = 2$ cannot be higher than the one chosen in $t = 1$.

In the second period, there are two possible states of the world. With probability π, the true state is revealed before the decision in $t = 2$ is taken by the DM. With probability $1-\pi$, the DM does not know the true state of the world when he chooses the optimal level of c_2 in $t = 2$: this state will be revealed after this decision has been made. We indicate with b_2^j the marginal benefit deriving from preservation in period 2, when the revealed state of the world is s^j, $j = u, f$. The probability distribution over the states of the world is $\left(s^u, p; s^f, 1-p\right)$ [20]. We also assume the benefit from preservation is negative if the state of the world is s^u ($u$nfavorable state), and positive if the state of the world is s^f ($f$avorable state), i.e. $b_2^u < 0, b_2^f > 0$ [21].

According to our assumptions, we indicate with:

c_2: amount of environmental resource preserved in $t = 2$, when the true state of the world has not been revealed before;

c_2^u: amount of environmental resource preserved in $t = 2$, when the DM knows the true state of the world is $s^j, j = u, f$.

A more intuitive way to identify the decision problem described above is to sum up the sequence of events through four main steps:

- Step (a). The DM chooses the amount of the environmental resource he wants to preserve in $t = 1$ (until $t = 2$).

- Step (b). Either the true state of the world is revealed or it is not.

The members of Θ are the various pairs of benefits and costs which could possibly accrue during the first period and second period for each possible decision which could be made. In that case, also benefits in period $t = 1$ are not known when choosing the level of preservation c_1.

The components of the pairs in Θ, b_t, can be thought of as four dimensional vectors (pc_t, pb_t, dc_t, db_t) representing preservation costs, preservation benefits, development costs and development benefits associated with the action taken in period t.

[19] We chose not to contemplate into the analysis the case $b_1 = 0$, since it makes the choice of c_1 irrelevant for what concerns benefits received in the first period. The same reasoning holds for b_2 (with respect to c_2).

[20] p is the probability of the state s^u before this state is revealed.

[21] Differently form Beltratti, Chichilnisky and Heal (1996), we do not assume that the expected benefit of preservation in the second period is positive, i.e. $pb_2^u + (1-p)b_2^s > 0$. We allow this quantity to be greater, equal or less than zero.

- Step (c). The DM chooses the amount of the environmental resource he wants to preserve in $t = 2$.

- Step (d). If in Step (b) no information has come out, now the true state of the world is revealed.

The structure of the decision problem is summarized in *Figure* 1.

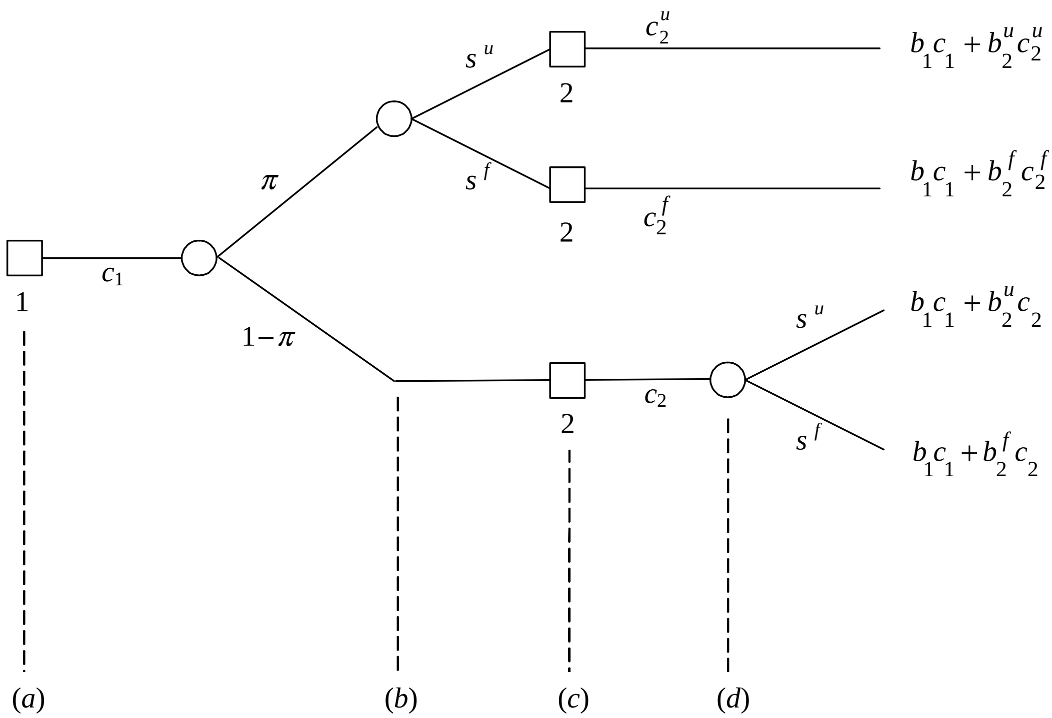


Figure 1

Before starting the analysis of the optimal preservation choice under the different information scenarios, it is useful to state a pair of results which holds independently from the kind of information structure we deal with (i.e., independently from the way in which π is defined):

Result 1. If in Step (b) the true state of the world is revealed, then

$$\begin{aligned} (c_2^u)^* &= 0, \\ (c_2^f)^* &= c_1. \end{aligned}$$

Result 2. If in Step (b) the true state of the world is not revealed, then

$$c_2{}^* = \begin{cases} 0 & \text{if} \quad pb_2^u + (1-p)b_2^s < 0 \\ c_1 & \text{if} \quad pb_2^u + (1-p)b_2^s > 0 \end{cases}$$

2.2 Modelling uncertainty

In our framework, uncertainty characterizes Step (b): the DM does not know if the true state of the world will be known or not when he takes his choice c_2 in Step (c). The key parameter is $\pi \in [0,1]$, the probability that the true state is revealed before choosing c_2, i.e. the probability new (complete) information arrives before Step (c).

According to the way in which we define the form and the properties of π, we are able to:

- allow for different "degrees of certainty" of the coming of new information: information can arrive with certainty ($\pi = 1$), with a certain probability ($\pi \in (0,1]$) or may not come out with certainty ($\pi = 0$);

- identify different kinds of new "information", according to its nature (looking at the components inside π). Information can be (only) exogenous, (only) endogenous, or both:

• in the first case, π does not depend on $(1-c_1)$, the amount of environmental resource developed in (and, obviously, on c_1, the amount of environmental resource preserved in) Step (a);

• in the second case, π does depend on c_1, or, rather, on $(1-c_1)$, the level of development. In particular, we assume that (as stated in Section 1.1.2), *in case of dependent learning, the level of information coming out is directly proportional to the level of development carried out*;

• in the third case, one part of information arrives exogenously and the rest comes out according to $(1-c_1)$; hence, $\pi = q + \lambda f(1-c_1)$, with $f'(\cdot) > 0$, $\lambda \in [0,1]$ being the probability of acquiring endogenous information (of the specified form) and $q \in [0,1]$ being the probability of acquiring exogenous information. In particular, we analyze the case where $f(1-c_1) = 1-c_1$, i.e. it is a linear (decreasing) function of c_1.

We summarize all information categories described above in a general case (and we call it *Case A*), then derive all the other subcases by imposing certain restrictions on the key parameters:

CASE A: Complete (i.e. Exogenous and Endogenous) information

$$\begin{aligned}\pi &= q + \lambda(1-c_1) \qquad \text{for} \quad c_1 \in [0,1] \\ &\quad \text{with} \quad q \in [0,1]\,, \quad \lambda \in [0, 1-q]\,.\end{aligned}$$

CASE B: (Only) Exogenous information

$$\textit{CASE A} \text{ with } \lambda = 0 \qquad \text{for} \quad c_1 \in [0,1]$$

CASE C: (Only) Endogenous information

$$\textit{CASE A} \text{ with } q = 0 \qquad \text{for} \quad c_1 \in [0,1]$$

CASE D: No information

$$\lambda = q = 0 \qquad \text{for} \quad c_1 \in [0,1]$$

The subcases "*information arriving with certainty*" [22] can be derived in the following ways:
- *Case A* with $\lambda = 1 - q$ (Complete info arriving with certainty, if $c_1 = 0$);
- *Case B* with $q = 1$ (Exogenous info arriving with certainty);
- *Case C* with $\lambda = 1$ (Endogenous info arriving with certainty, if $c_1 = 0$).

3 Maximization Problem and Optimal Choices

In this section, we write down the DM's maximization problem and find the optimal choices c_1^*, c_2^* in each of the four information structures described in Section 2.2.

3.1 *Case A*: Complete (i.e. Endogenous and Exogenous) Information

Let us write and solve the DM's utility maximization problem in the most general case, in which both exogenous and endogenous information are available with a certain probability (respectively, $q \in [0,1]$ and $\lambda \in [0,1]$) after the decision made in $t = 1$, i.e. at Step (b).

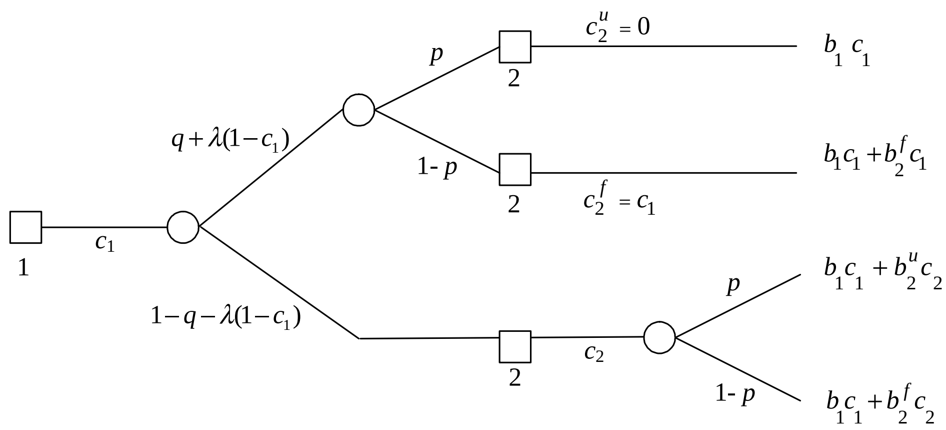


Figure 2

[22] We analyze this case separately only because it is the most frequently analyzed in environmental option value literature: thus, it allows us to make comparisons and to show that our results hold even under the restriction of information arriving with certainty.

Given *Result* 1 and *Result* 2, the realized payoffs are those indicated in *Figure* 2.

The *DM*'s expected value of net benefits of preservation in both periods (given *Result* 1 and *Result* 2) is

$$\begin{aligned} EV(c_1,(c_2^u)^*,(c_2^f)^*,c_2) &= [q+\lambda(1-c_1)]\left[b_1+(1-p)b_2^f\right]c_1 + \\ &\quad \{1-[q+\lambda(1-c_1)]\}\left[b_1c_1+pb_2^uc_2+(1-p)b_2^fc_2\right] \end{aligned}$$

By analyzing the "low part" of the compound lottery in *Figure* 2, one can distinguish among two cases, according to the expected value of the second-period net benefits when the *DM* does not receive new information at Step (b):

(i) $\quad pb_2^u+(1-p)b_2^f<0$

The optimal levels of preservation in $t=1$ and in $t=2$ are:

$$c_1{}^* = \begin{cases} 0 & \text{if } b_1\in\left[-\infty,-(1-p)(\lambda+q)b_2^f\right] & (i)' \\ \frac{b_1+(1-p)(q+\lambda)b_2^f}{2(1-p)\lambda b_2^f} & \text{if } b_1\in\left[-(1-p)(\lambda+q)b_2^f,(1-p)(\lambda-q)b_2^f\right] & (i)'' \\ 1 & \text{if } b_1\in\left[(1-p)(\lambda-q)b_2^f,0\right] \quad ,\lambda\le q & (i)''' \end{cases}$$
$$c_2{}^* = 0$$

The optimal expected value function $EV^*(b_1,b_2^u,b_2^f)$ is

$$EV^* = \begin{cases} 0 & \text{if } b_1\in\left[-\infty,-(1-p)(\lambda+q)b_2^f\right] & (i)' \\ \frac{\left[b_1+(1-p)(q+\lambda)b_2^f\right]^2}{4(1-p)\lambda b_2^f} & \text{if } b_1\in\left[-(1-p)(\lambda+q)b_2^f,(1-p)(\lambda-q)b_2^f\right] & (i)'' \\ b_1+(1-p)(q+\lambda)b_2^f & \text{if } b_1\in\left[(1-p)(\lambda-q)b_2^f,0\right] \quad ,\lambda\le q & (i)''' \end{cases}$$

(ii) $\quad pb_2^u+(1-p)b_2^f>0$

The optimal levels of preservation in $t=1$ and in $t=2$ are:

$$c_1{}^* = \begin{cases} 0 & \text{if } b_1\in\left[-\infty,-(1-p)(\lambda+q)b_2^f\right] & (i)' \\ \frac{b_1+(1-p)(q+\lambda)b_2^f}{\lambda\left[(1-p)b_2^f-pb_2^u\right]} & \text{if } b_1\in\left[-(1-p)(\lambda+q)b_2^f,(1-p)(\lambda-q)b_2^f\right] & (i)'' \\ 1 & \text{if } b_1\in\left[(1-p)(\lambda-q)b_2^f,0\right] \quad ,\lambda\le q & (i)''' \end{cases}$$
$$c_2{}^* = c_1{}^*$$

The optimal expected value function $EV^*(b_1, b_2^u, b_2^f)$ is [23]

$$EV^* = \begin{cases} 0 & \text{if } \; b_1 + (1-p)b_2^f + (1-q)b_2^u \in [-\infty, \lambda b_2^u] & (ii)' \\ -\frac{\left[b_1+(1-p)b_2^f+p(1-q-\lambda)b_2^u\right]^2}{4p\lambda b_2^u} & \text{if } \; b_1 + (1-p)b_2^f + (1-q)b_2^u \in [\lambda b_2^u, -\lambda b_2^u] & (ii)'' \\ b_1+(1-p)b_2^f+p(1-q)b_2^u & \text{if } \; b_1 + (1-p)b_2^f + (1-q)b_2^u \in [-\lambda b_2^u, +\infty] & (ii)''' \end{cases}$$

3.2 *CASE B*: (Only) Exogenous Information

When $\lambda = 0$ (no endogenous information) the DM in *Step* (a) knows that at *Step* (b) with probability $q \in [0, 1]$ he will know the realized value of the net benefit b_2^j. Hence, he knows that information coming is independent from the preservation level chosen in *Step* (a), although in the same step he is not sure that he will know the true state of the world when he will choose at *Step* (c).

We solve the DM's utility maximization problem by applying the same principle of reduction of the compound lotteries and the same procedure of maximization we use in *Case A*.

Given *Result* 1 and *Result* 2, the realized payoffs are those indicated in *Figure* 2, taking into account the probabilities assigned to the branches of the trees are different (we have q in place of $q + \lambda(1 - c_1)$).

Taking into account *Result* 1 and *Result* 2, the DM's expected payoff is

$$\begin{aligned} EV_{exo}(c_1, (c_2^u)^*, (c_2^f)^*, c_2) &= q\left[(pb_1 + (1-p)(b_1 + b_2^f)\right] c_1 + \\ &\quad +(1-q)\left[p(b_1c_1 + b_2^u c_2) + (1-p)(b_1c_1 + b_2^f c_2)\right] \\ &= q\left[b_1 + (1-p)b_2^f\right] c_1 + (1-q)\left[b_1c_1 + (pb_2^u + (1-p)b_2^f)c_2\right] \end{aligned}$$

(i) If $pb_2^u + (1-p)b_2^f < 0$, the optimal levels of preservation in the two periods are

$$\begin{aligned} {c_1}^* &= \begin{cases} 0 & \text{if } \; b_1 + q(1-p)b_2^f < 0 & (i)' \\ 1 & \text{if } \; b_1 + q(1-p)b_2^f > 0 & (i)'' \end{cases} \\ {c_2}^* &= 0 \end{aligned}$$

[23] When $q+\lambda \in [0, 1)$, the DM at *Step* (a) knows that, even if he decides to destroy the entire environmental resource ($c_1 = 0$), he is not certain that at *Step* (b) the true state of the world will come out. Hence, even if he destroys everything in $t = 1$, he is not sure that he will know the realized value of the net benefit b_2^i when choosing c_2 at *Step* (c). On the contrary, when $q + \lambda = 1$, the DM is sure that, in case he destroys everything ($c_1 = 0$) he will receive with certainty some information at *Step* (b). We call this subcase "Complete information arriving with certainty, if $c_1 = 0$". The optimal values for c_1 and c_2 and the optimal expected value of net benefits over both periods can be easily found by substituting $q + \lambda = 1$ into the results obtained for the general case.

consequently the optimal expected value function $EV^*_{exo}(b_1, b_2^u, b_2^f)$ is

$$EV^*_{exo} = \begin{cases} 0 & \text{if } \; b_1 + q(1-p)b_2^f < 0 \quad (i)' \\ b_1 + q(1-p)b_2^f & \text{if } \; b_1 + q(1-p)b_2^f < 0 \quad (i)'' \end{cases}$$

(ii) If $pb_2^u + (1-p)b_2^f > 0$, the optimal levels of preservation in the two periods are

$$\begin{aligned} c_1^* &= \begin{cases} 0 & \text{if } \; b_1 + (1-q)pb_2^u + (1-p)b_2^f < 0 \quad (ii)' \\ 1 & \text{if } \; b_1 + (1-q)pb_2^u + (1-p)b_2^f > 0 \quad (ii)'' \end{cases} \\ c_2{}^* &= c_1{}^* \end{aligned}$$

and the optimal expected value function $EV^*_{exo}(b_1, b_2^u, b_2^f)$ is

$$EV^*_{exo} = \begin{cases} 0 & \text{if } \; b_1 + (1-q)pb_2^u + (1-p)b_2^f < 0 \quad (ii)' \\ b_1 + (1-q)pb_2^u + (1-p)b_2^f & \text{if } \; b_1 + (1-q)pb_2^u + (1-p)b_2^f > 0 \quad (ii)'' \end{cases}$$

A particular subcase: Exogenous Information arriving with certainty

This is the case in which the "standard" *QOV à la* Arrow-Fisher (as analyzed in Section 1.1) emerges:.if new exogenous information arrives with certainty ($q = 1$) in *Step* (b), the *DM* at *Step* (a) knows that when deciding at *Step* (c) he will know if the net benefit is b_2^u or b_2^f.
Hence, the decision problem in *Figure* 1 can be reduced to the one in *Figure* 6 below.

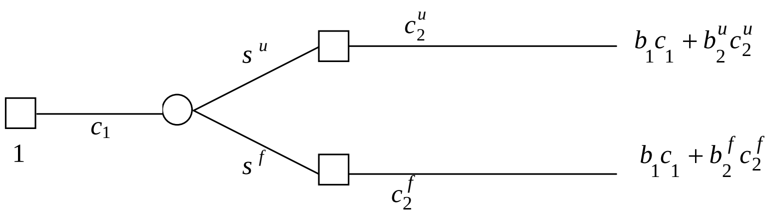


Figure 6

The expected value of the lottery is

$$EV_{exo|q=1}(c_1, c_2^u, c_2^f) = b_1c_1 + pb_2^uc_2^u + (1-p)b_2^fc_2^f$$

Given *Result* 1,

$$EV_{exo|q=1}(c_1, (c_2^u)^*, (c_2^f)^*) = b_1c_1 + (1-p)b_2^fc_1$$

Since the expected value function is linear in c_1, the optimal level of preservation in $t = 1$ is

$$\begin{array}{lll} {c_1}^* = 0 & \text{if} \quad b_1 + (1-p)b_2^f < 0 & (i) \\ {c_1}^* = 1 & \text{if} \quad b_1 + (1-p)b_2^f > 0 & (ii) \end{array}$$

and the optimal expected value function $EV_{exo|q=1}(b_1, b_2^u, b_2^f)$ is

$$EV_{exo|q=1} = \begin{cases} 0 & \text{if} \quad b_1 + (1-p)b_2^f < 0 \quad (i) \\ b_1 + (1-p)b_2^f & \text{if} \quad b_1 + (1-p)b_2^f > 0 \quad (ii) \end{cases}$$

3.3 *Case C*: (Only) Endogenous Information

The optimal values of c_1, c_2 and of the expected benefits in case only endogenous information is possible can be easily derived by writing down results found in *Case A* and imposing $q = 0$ (no exogenous information).

(i) If $pb_2^u + (1-p)b_2^f < 0$, it can never be $b_1 > (1-p)\lambda b_2^f$, thus the region $(i)'''$ (in which ${c_1}^* = 1$) disappears (i.e. it will never be ${c_1}^* = 1$ in the subcase (i)), and the optimal levels of preservation in the two periods are

$$\begin{aligned} {c_1}^* &= \begin{cases} 0 & \text{if} \quad b_1 \in \left[-\infty, -(1-p)\lambda b_2^f\right] \quad (i)' \\ \frac{b_1+(1-p)\lambda b_2^f}{2(1-p)\lambda b_2^f} & \text{if} \quad b_1 \in \left[-(1-p)\lambda b_2^f, 0\right] \quad (i)'' \end{cases} \\ {c_2}^* &= 0 \end{aligned}$$

and thus the optimal expected value function $EV^*_{endo}(b_1, b_2^u, b_2^f)$ is

$$EV^*_{endo} = \begin{cases} 0 & \text{if} \quad b_1 \in \left[-\infty, -(1-p)\lambda b_2^f\right] \quad (i)' \\ \frac{\left[b_1+(1-p)\lambda b_2^f\right]^2}{4(1-p)\lambda b_2^f} & \text{if} \quad b_1 \in \left[-(1-p)\lambda b_2^f, 0\right] \quad (i)'' \end{cases}$$

(ii) If $pb_2^u + (1-p)b_2^f > 0$, the optimal levels of preservation in the two periods are

$$\begin{aligned} {c_1}^* &= \begin{cases} 0 & \text{if} \quad b_1 + (1-p)b_2^f + b_2^u \in [-\infty, \lambda b_2^u] \quad (ii)' \\ -\frac{b_1+(1-p)b_2^f+p(1-\lambda)b_2^u}{2p\lambda b_2^u} & \text{if} \quad b_1 + (1-p)b_2^f + b_2^u \in [\lambda b_2^u, -\lambda b_2^u] \quad (ii)'' \\ 1 & \text{if} \quad b_1 + (1-p)b_2^f + b_2^u \in [-\lambda b_2^u, +\infty] \quad (ii)''' \end{cases} \\ {c_2}^* &= {c_1}^* \end{aligned}$$

and thus the optimal expected value function $EV^*_{endo}(b_1, b_2^u, b_2^f)$ is

$$EV^*_{endo} = \begin{cases} 0 & \text{if} \quad b_1 + (1-p)b_2^f + b_2^u \in [-\infty, \lambda b_2^u] \quad (ii)' \\ -\frac{\left[b_1+(1-p)b_2^f+p(1-q-\lambda)b_2^u\right]^2}{4p\lambda b_2^u} & \text{if} \quad b_1 + (1-p)b_2^f + b_2^u \in [\lambda b_2^u, -\lambda b_2^u] \quad (ii)'' \\ b_1 + (1-p)b_2^f + pb_2^u & \text{if} \quad b_1 + (1-p)b_2^f + b_2^u \in [-\lambda b_2^u, +\infty] \quad (ii)''' \end{cases}$$

A particular subcase: Endogenous Information arriving with certainty, if $c_1 = 0$

The case $\lambda = 1$ serves the aim of comparing our results with those common in environmental option values literature, in which the most representative framework of environmental decisions under uncertainty and irreversibility is that of Freeman (1984) and Fisher and Hanemann (1987): within this model, in the information scenario where information is completely endogenous ($q = 0$), it arrives with certainty ($\lambda = 1$). Under these two restrictions, we could compare our TV to the "standard" QOV_ε *à la* Arrow-Fisher (as analyzed in Section 1.2).
The optimal values for c_1 and c_2 and the optimal expected value of net benefits over both periods can be easily found by substituting $\lambda = 1$ in the results obtained for the general case (*Case A*).

3.4 *Case D*: No information

The decision problem represented in *Figure* 1 is reduced to that in *Figure* 6 below.

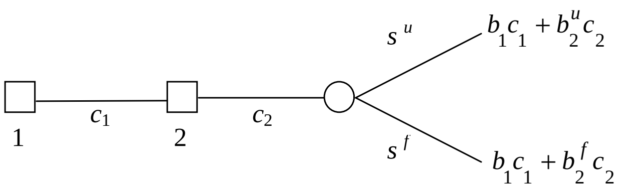


Figure 6

The expected value of the lottery is

$$EV(c_1, c_2) = b_1c_1 + \left[pb_2^u + (1-p)b_2^f\right] c_2$$

Let us write the expected value of net benefits of preservation as a function of c_1 only, by choosing c_2 optimally in the second period: by this way, we obtain the expected value of preserving in *Step* (a), given that the DM's choice in *Step* (c) is optimal.

By looking at *Result* 2,

(i) $\quad pb_2^u + (1-p)b_2^f < 0 \quad \Longrightarrow \quad {c_2}^* = 0$

$\Longrightarrow \quad EV_{no}(c_1, {c_2}^* = 0) = b_1c_1$

By maximizing with respect to c_1,

$$c_1{}^* = c_2{}^* = 0$$

thus the optimal expected value function is

$$EV_{no}{}^*(b_1, b_2^u, b_2^f) = 0$$

$$(ii) \quad pb_2^u + (1-p)b_2^f > 0 \quad \Longrightarrow \quad c_2{}^* = c_1$$

$$\Longrightarrow \quad EV_{no}(c_1, c_2{}^* = c_1) = \left[b_1 + pb_2^u + (1-p)b_2^f\right] c_1.$$

By maximizing,

$$\begin{aligned} c_1{}^* &= \begin{cases} 0 & \text{if} \quad b_1 + pb_2^u + (1-p)b_2^f < 0 \quad (ii)' \\ 1 & \text{if} \quad b_1 + pb_2^u + (1-p)b_2^f > 0 \quad (ii)'' \end{cases} \\ c_2{}^* &= c_1{}^* \end{aligned}$$

thus the optimal expected value function $EV_{no}{}^*(b_1, b_2^u, b_2^f)$ is

$$EV_{no}{}^* = \begin{cases} 0 & \text{if} \quad b_1 + pb_2^u + (1-p)b_2^f < 0 \quad (ii)' \\ b_1 + pb_2^u + (1-p)b_2^f & \text{if} \quad b_1 + pb_2^u + (1-p)b_2^f > 0 \quad (ii)'' \end{cases}$$

Noting that condition $(ii)''$ implies (ii) and that condition (i) and (ii) are complementary, we can sum up the optimal choices in the two subcases:

$$\begin{array}{ll} c_1{}^* = c_2{}^* = 0 & \text{if} \quad b_1 + pb_2^u + (1-p)b_2^f < 0 \\ c_1{}^* = c_2{}^* = 1 & \text{if} \quad b_1 + pb_2^u + (1-p)b_2^f > 0 \end{array}$$

and the relative optimal expected value function $EV_{no}{}^*(b_1, b_2^u, b_2^f)$ is

$$EV_{no}{}^* = \begin{cases} 0 & \text{if} \quad b_1 + pb_2^u + (1-p)b_2^f < 0 \\ b_1 + pb_2^u + (1-p) & \text{if} \quad b_1 + pb_2^u + (1-p)b_2^f > 0 \end{cases}$$

4 Calculus of the WV and of the TV

In this section, we use the results of the DM's maximization problem (the optimal level of c_1 and c_2 and the optimal expected value functions in each of the four information structures) in order to express the WV and the TV as functions of the parameters of the decision problem and describe their properties and their effects on DM's optimal behaviour.

4.1 Graphical representation of the optimal preservation choices

First of all, let us represent graphically the results we have obtained on the optimal level of c_1 and c_2 and on the expected value functions $EV(\cdot)$. We introduce a Cartesian plane with the relative benefits $-\frac{b_1}{b_2^f}$ on the x-axis and the relative benefits $-\frac{b_2^u}{b_2^f}$ on the y-axis. [24] Obviously, when these two ratios vary (when the values of the net benefits of preservation in $t = 1$ and $t = 2$ vary), the optimal preservation choices for the first and for the second period change too. In *Table* 1, we indicate for each figure the information scenario represented inside and the specific values of the three parameters p, q and λ.

Figure A.1	*Case A* with $p = \frac{1}{2}, q = \frac{1}{3}, \lambda = \frac{1}{3}$
Figure A.2	*Case A* with $p = \frac{1}{2}, q = \frac{1}{3}, \lambda = \frac{1}{4}$
Figure A.3	*Case A* with $p = \frac{1}{2}, q = \frac{1}{3}, \lambda = \frac{2}{3}$
Figure A.4	*Case A* with $p = \frac{1}{2}, q = \frac{2}{3}, \lambda = \frac{1}{3}$
Figure B.1	*Case B* with $p = \frac{1}{2}, q = \frac{1}{3}$
Figure B.2	*Case B* with $p = \frac{1}{2}, q = 1$
Figure C	*Case C* with $p = \frac{1}{2}, \lambda = 1$
Figure D	*Case D* with $p = \frac{1}{2}$

(*Table* 1. Cases and parameters relative to each *Figure*.)

A brief description of the preservation choice path in each of the different colorful regions of the quadrant I of the Cartesian plane:

• "*White*" region: the DM does not preserve anything in $t = 1$ and so also in $t = 2$;

• "*Green*" region: the DM preserves everything both in $t = 1$ and also in $t = 2$;

• "*Yellow*" region: the DM preserves only a part of the resource in $t = 1$ and the same amount in $t = 2$;

• "*Blue*" region: the DM preserves everything in $t = 1$ and nothing in $t = 2$;

• "*Orange*" region: the DM preserves only a part of the resource in $t = 1$ and nothing in $t = 2$;

[24] Our choice of these two ratios as variables for the Cartesian axes can be explained, among others, by the need of working with positive quantities (since $b_1, b_2^u < 0, b_2^f > 0$), in order to concentrate in quadrant I of the Cartesian plane the analysis of the conditions (inequalities) we have found by solving the maximization problem.

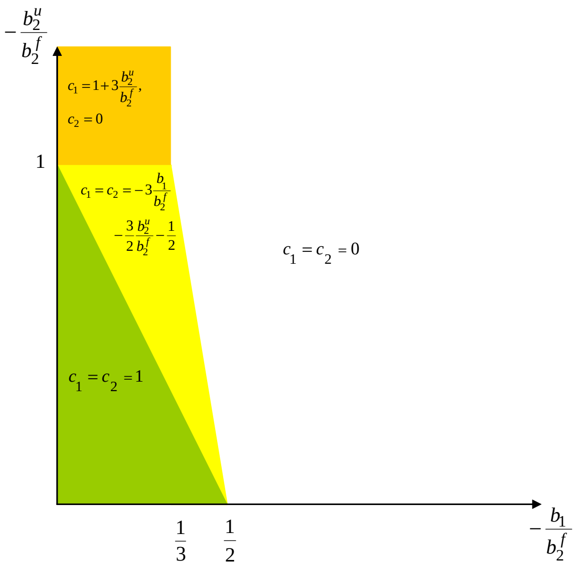


Figure A.1

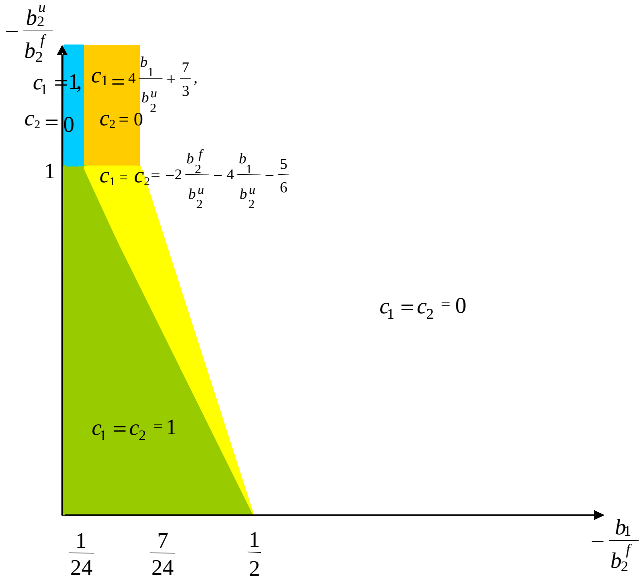


Figure A.2

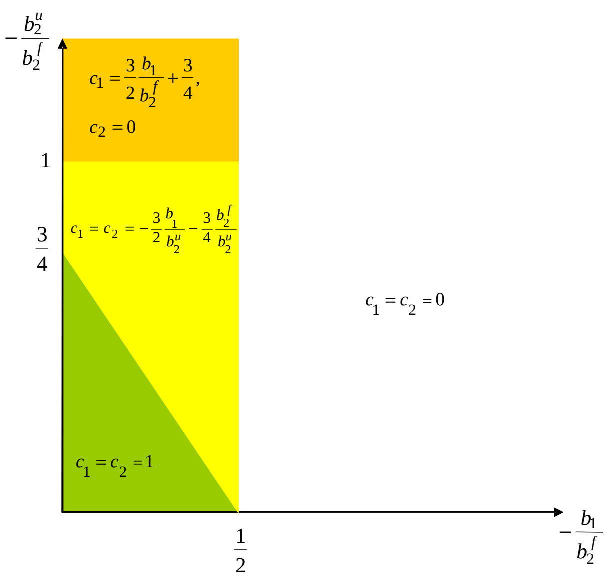


Figure A.3

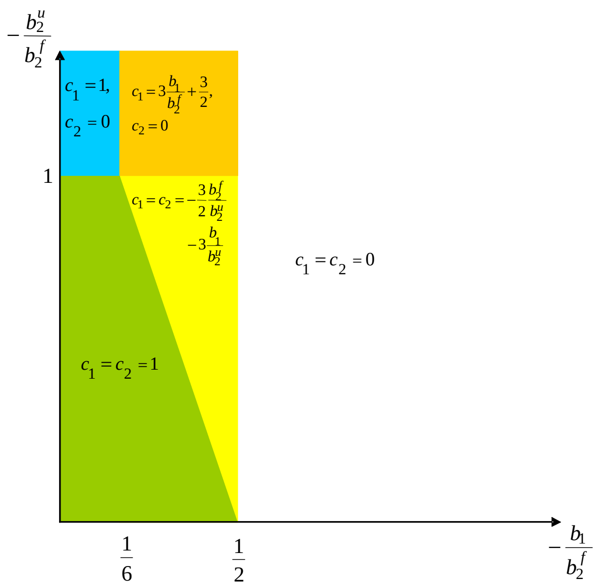


Figure A.4

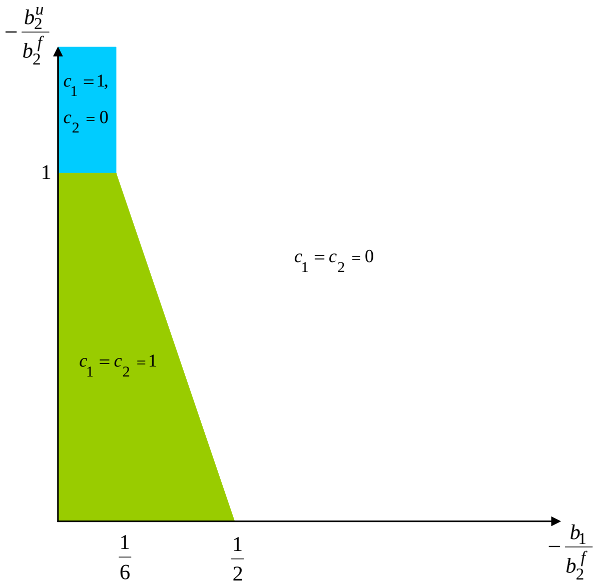


Figure B.1

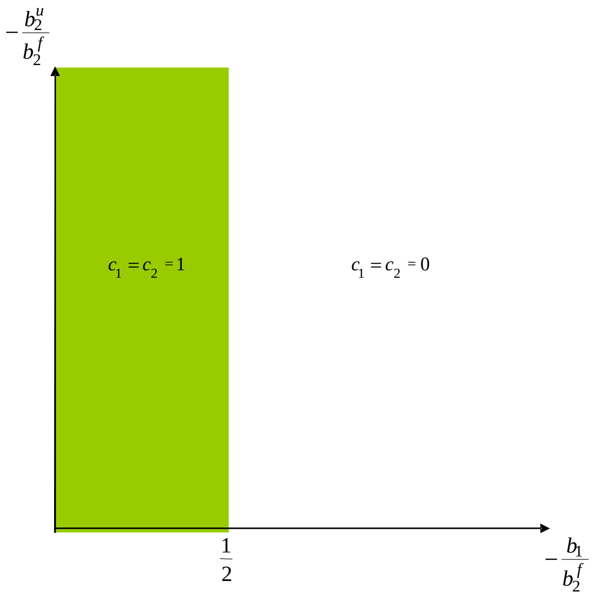


Figure B.2

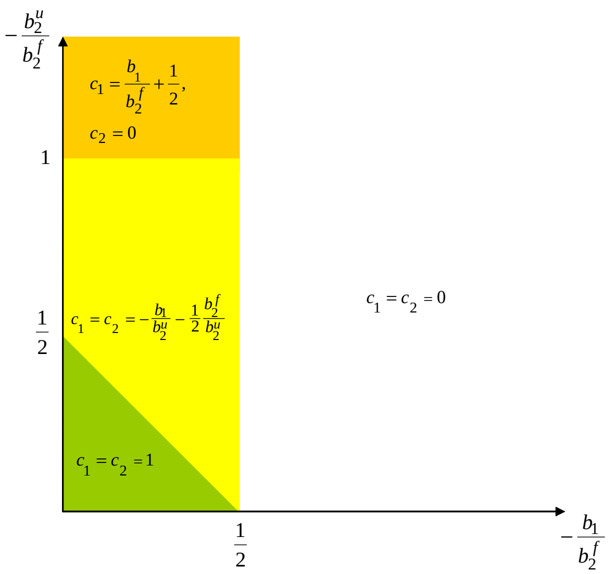


Figure C

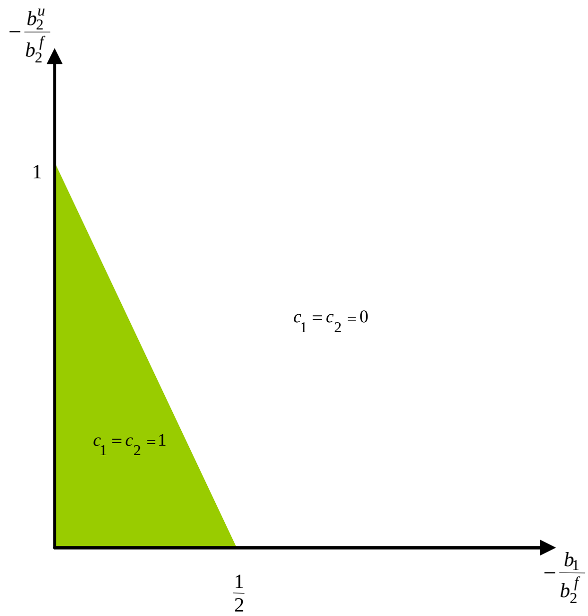


Figure D

4.2 Properties of the Waiting Value

Let us calculate the Waiting Value using the expression we have introduced in Section 1.1, i.e.

$$WV = EV_{exo}{}^* - EV_{no}{}^*$$

Since $EV_{exo}{}^*$ and $EV_{no}{}^*$ vary according to the values of b_1, b_2^u and b_2^f, we have to calculate the difference between the two optimal expected values by looking at the different regions we identify in the Cartesian plane $\left(-\frac{b_1}{b_2^f}, -\frac{b_2^u}{b_2^f}\right)$: we do it by comparing the optimal expected values in $Figure\ B.1$ to those in $Figure\ D$.

· For $-\frac{b_2^u}{b_2^f} > 1$ and $-\frac{b_1}{b_2^f} > 1, \qquad WV = 0.$

· For $-\frac{b_2^u}{b_2^f} < 1$ and $-\frac{b_2^u}{b_2^f} > \frac{1-p}{(1-q)p} - \frac{1}{(1-q)p}\left(-\frac{b_1}{b_2^f}\right), \qquad WV = 0.$

· For $-\frac{b_2^u}{b_2^f} > 1$ and $-\frac{b_1}{b_2^f} > q(1-p), \qquad WV = b_1 + q(1-p)b_2^f > 0.$

· For $\frac{p}{1-p} - \frac{1}{1-p}\left(-\frac{b_1}{b_2^f}\right) < -\frac{b_2^u}{b_2^f} < 1$ and $-\frac{b_2^u}{b_2^f} < \frac{1-p}{(1-q)p} - \frac{1}{(1-q)p}\left(-\frac{b_1}{b_2^f}\right),$
$WV = b_1 + (1-q)pb_2^u + (1-p)b_2^f > 0.$

· For $-\frac{b_2^u}{b_2^f} < \frac{p}{1-p} - \frac{1}{1-p}\left(-\frac{b_1}{b_2^f}\right), \qquad WV = -qpb_2^u > 0.$

Thus, the WV is increasing in the probability of receiving new information (exogenously) and in the level of the net benefits in the favorable state; it is decreasing both in the level of the net benefits in the current period (considered in absolute value, given that they are negative by assumption) and in the level of the net benefits in the unfavorable state (taken in absolute value and given that they are negative by assumption). For what concerns the *a priori* probability of the state s^u (unfavorable state) before this state is revealed, the WV is decreasing in p in "unfavorable" regions (i.e. regions where $|b_1|$ and/or $|b_2^u|$ are very high and/or b_2^f is very low) and increasing in p in "favorable" regions (defined in the opposite way). Briefly,

$$WV = WV\left(\underset{+,-}{p}, \underset{+}{q}, \underset{-}{|b_1|}, \underset{-}{|b_2^u|}, \underset{+}{b_2^f} \right)$$

If following the literature on quasi-option values we would calculate the WV as the difference between the optimal expected value in case of *certain* exogenous information and the optimal expected value in the no information case, i.e.

$$WV_{|q=1} = EV_{exo|q=1}{}^* - EV_{no}{}^* \geq 0$$

we would find an upper bound for the WV we have calculated above, i.e. $WV_{|q=1} \geq WV$, since WV is increasing in q.

Our conclusions on the properties of the WV are more general and "robust" than those of the standard literature on quasi-option values: we find that *in absence of endogenous information, even though exogenous information does not arrive with certainty* (*but with a given probability* $q \in (0,1)$), *the WV is always non-negative, thus forcing the DM towards a higher level of preservation of the environmental area during the first and the second period of choice.*

In fact, looking at the results shown in $Figure\ B.1$ and D, it is not difficult to notice that in all regions of the quadrant I of the Cartesian plane it is always

$$\begin{array}{rclr} (c_1{}^*)_{exo} & \geq & (c_1{}^*)_{no} & (a) \\ (c_2{}^*)_{exo} & \geq & (c_2{}^*)_{no} & (b) \end{array}$$

Result (a) is obvious.

Result (b) can be proved by applying this reasoning: because of irreversibility, $(c_2{}^*)_{exo} \leq (c_1{}^*)_{exo}$, $(c_2{}^*)_{no} \leq (c_1{}^*)_{no}$, but since $(c_1{}^*)_{exo} \geq (c_1{}^*)_{no}$ in Step (c) the DM has a larger choice set from which choosing $(c_2{}^*)_{exo}$; since the choice $(c_2{}^*)_{no}$ is possible also in case of exogenous information (because it is surely $(c_1{}^*)_{exo} \geq (c_1{}^*)_{no}$), being objective function the same under each information structure, $(c_2{}^*)_{exo}$ cannot be lower than $(c_2{}^*)_{no}$.

4.3 Properties of the Testing Value

According to the definition we have introduced in Section 1.2, we define the Testing value as

$$TV = EV^* - EV^*_{exo}$$

Hence,

$$EV^* - EV^*_{no} = WV + TV$$

We could calculate the TV not (only) as an *additional* value of endogenous to exogenous information (as we did above), rather as a value emerging in the particular information context in which *only* endogenous information is available, i.e.

$$TV' = EV^*_{endo} - EV_{no}$$

In case the utility function would be linear (and in our case it is), one could easily verify that $TV = TV'$.

By applying the latter method, one could get an environmental value (linked to endogenous information) closer to the QOV_{ε}, as defined by Freeman (1984) and Fisher and Hanemann (1987), i.e. the difference between the expected benefits in case of *certain* endogenous information and the expected benefits in the no information case:

$$TV'_{|\lambda=1} = EV^*_{endo|\lambda=1} - EV_{no}$$

The $TV'_{|\lambda=1}$ represents a base of comparison between our results and those coming from the standard literature, that we have exhibited in Section 1.2. Nonetheless, assuming again that the DM's utility function is linear, through the $TV'_{|\lambda=1}$ we could easily derive the upper bound for the TV, i.e. $TV'_{|\lambda=1} \geq TV' = TV$, since the TV is increasing in λ.

Now, let's turn to the first definition of TV, the one we have introduced *ex novo* in this thesis chapter. Since EV^* and $EV_{exo}{}^*$ vary according to the values of b_1, b_2^u and b_2^f, we have to calculate the difference between the two optimal expected values in the different regions of the Cartesian plane $\left(-\frac{b_1}{b_2^f}, -\frac{b_2^u}{b_2^f}\right)$ under the "both exogenous and endogenous" information scenario and under the "only exogenous" information scenario, respectively.

We calculate the TV for different values of λ and q , for two reasons:

- to show that our results hold independently from the values one can assign to λ and q; [25]

- to analyze the behavior of the TV as a function of λ and q.

Let us first compare $Figure\ A.3$ to $Figure\ B.1$ and calculate the $TV(b_1, b_2^u, b_2^f)$ in case $p = \frac{1}{2}, q = \frac{1}{3}$ and $\lambda = \frac{1}{3}$.

For $-\frac{b_2^u}{b_2^f} > 1$ and $-\frac{b_1}{b_2^f} > \frac{1}{3}$, $\quad TV = 0.$

For $-\frac{b_2^u}{b_2^f} > 1$ and $\frac{1}{6} < -\frac{b_1}{b_2^f} < \frac{1}{3}$, $\quad TV = \frac{3}{2}\frac{\left(b_1+\frac{1}{3}b_2^f\right)^2}{b_2^f} > 0.$

For $-\frac{b_2^u}{b_2^f} > 1$ and $0 < -\frac{b_1}{b_2^f} < \frac{1}{6}$, $\quad TV = \frac{3}{2}\frac{(b_1)^2}{b_2^f} > 0.$

For $-\frac{b_2^u}{b_2^f} < 1$ and $-\frac{b_2^u}{b_2^f} > 3 - 6\left(-\frac{b_1}{b_2^f}\right)$, $\quad TV = 0.$

For $-\frac{b_2^u}{b_2^f} < 1, -\frac{b_2^u}{b_2^f} < 3 - 6\left(-\frac{b_1}{b_2^f}\right)$ and $-\frac{b_2^u}{b_2^f} > \frac{3}{2} - 3\left(-\frac{b_1}{b_2^f}\right)$,

$TV = -\frac{3\left(b_1+\frac{1}{2}b_2^f+\frac{1}{6}b_2^u\right)^2}{2b_2^u} > 0.$

For $-\frac{b_2^u}{b_2^f} < 1, -\frac{b_2^u}{b_2^f} < \frac{3}{2} - 3\left(-\frac{b_1}{b_2^f}\right)$ and $-\frac{b_2^u}{b_2^f} > 1 - 2\left(-\frac{b_1}{b_2^f}\right)$,

[25] We will show also that our results hold for every $p \in [0,1]$; but, since this is immediate (just look at the formulas we write down), we don't need to do any comparative statics analysis based on p.

$TV = -\frac{3}{2}\frac{\left(b_1+\frac{1}{2}b_2^f-\frac{1}{2}b_2^u\right)^2}{b_2^u} > 0.$

For $-\frac{b_2^u}{b_2^f} < 1-2\left(-\frac{b_1}{b_2^f}\right), \qquad TV = 0.$

Let us now compare $Figure\ A.4$ to $Figure\ B.1$ and calculate the $TV(b_1, b_2^u, b_2^f)$ in case $p=\frac{1}{2}$ and $\lambda=\frac{1}{4}<q=\frac{1}{3}$.

The only "relevant" difference with the case in which $\lambda \geq q$ is in the north-west of quadrant I of the Cartesian plane, where we can find a region s.t. even with endogenous information (additional to the exogenous one) the DM preserves everything in the first period ($(c_1^*)_{compl}=1$) and nothing in the second ($(c_2^*)_{compl}=0$). This region comes out only when $\lambda < q$; in other words, when it is more likely that new information at $Step\ (c)$ arrives exogenously than endogenously, . This happens only for $|b_1|$ very low and $|b_2^u|$ much higher than b_2^f.

For the values of the net benefits (b_1, b_2^u, b_2^f) belonging to this region, defined by the inequalities

$$\begin{cases} -\frac{b_2^u}{b_2^f} > 1 \\ -\frac{b_1}{b_2^f} < \frac{1}{24} \end{cases}$$

the Testing Value is again positive,

$$TV = (1-p)\lambda b_2^f = \frac{1}{8}b_2^f > 0.$$

From the discussion above and from a careful analysis of $Figures\ A.1$ - D, the main features of the Testing value turn out to be:

(1) $TV \geq 0$

(2) $TV = TV(\underset{+,-}{p}, \underset{-}{q}, \underset{+}{\lambda}, \underset{+,-}{|b_1|}, \underset{+,-}{|b_2^u|}, \underset{+}{b_2^f})$

(3.a). Given b_2^f and $|b_2^u|$, for high values of $|b_1|$ (but not so high), it happens that $(c_1^*)_{compl} > (c_1^*)_{exo}$.

(3.b) Given b_2^f, for high values of $|b_1|$ (but not so high) and low values of $|b_2^u|$ (but not so low), it happens that $(c_1^*)_{compl} > (c_1^*)_{exo}$ and $c_2^* = (\hat{c}_2)_{exo}$.

(4) The higher (lower) the value of λ (q), the larger the region in which $(c_1^*)_{compl} > (c_1^*)_{exo}$ and the region in which $(c_t^*)_{compl} > (\hat{c}_t)_{exo}$ $\forall t=1,2$. [26]

[26] As a final remark, we want to underline that we have verified that all the results about the optimal preservation levels, the WV and the TV we have presented in this paper hold also in case the DM is risk-adverse, i.e. if his utility function is concave; they hold independently from its concavity, i.e. independently from the DM's degree of risk aversion. Nonetheless, the results on the TV hold all the more so when the DM is risk-averse. Details on calculations are available upon request.

5 Conclusions

Our theoretical work presented in this first chapter of this thesis goes several steps beyond the existing literature on environmental option values.

First of all, we generalize the features and extend the application of the existing environmental choice framework.

More specifically, starting with the traditional two-period model of choice (develop or preserve), we allow for a continuous choice set and analyze the information side in the most complete possible way; in our model, we are able to identify:

- if information comes out with certainty, with a certain probability or if it does not;

- if information is (only) exogenous, (only) endogenous, or both.

In this more general framework, moving from the analysis of the meaning of the QOV, we have defined a more general $Waiting\ Value$ (WV) as the value attached to the increase in expected utility (of preservation and development net benefits) due to the possibility of acquiring new *information exogenously*. We have shown the WV (as the QOV) is always positive, thus forcing the DM towards a higher level of preservation of the environmental area during the first and the second period of choice.

In the environmental option values literature little attention has been devoted to the "*endogenous information*" (dependent learning) case. Our "general" framework allows for it.

There are several environmental choice problems needing to be modelled by accounting for the fact that by developing (destroying) even a little portion of an environmental resource, you are able to obtain (more) information on future net benefits of preservation; nonetheless, the level and the quantity of information coming out endogenously depends on the amount of the resource developed (destroyed).

Miller and Lad (1984), Freeman (1984), Hanemann and Fisher (1987) and again Fisher (2000) have shown that if information about the consequences of an irreversible development action can be obtained only by undertaking development, this strengthens the case for some development. In other words, allowing for the possibility of new information acquired by developing (destroying) at least a portion of the environmental resource would surely lead to a lower amount of preservation in both periods with respect to the case in which information comes out only exogenously. Their conclusion is really intuitive: the fact that "the more you destroy, the higher the possibility of obtaining new information" would increase dramatically the amount of resource developed in the first period.

In the second part of this chapter, we prove the *counterintuitive result* that (for a large set of values of net benefits of preservation) the possibility of endogenous information pushes the DM towards a higher level of preservation with respect to the case where information arrives only exogenously.

By generalizing Hanemann and Fisher's (1987) analyses, throughout this chapter of our thesis we have introduced a $Testing\ Value$ (TV), defined as the additional value attached to endogenous information (additional with respect to information exogenously arriving); in other words, it is the gain the DM obtains when he can receive information regarding future benefits, by developing in the current period (with respect to the case in which he ignores the possibility of receiving information in this manner). The need for a TV arises in all those situations in which development of an environmental area itself generates information about the future economic benefits of development (and its future environmental costs).

We have shown the TV too is always positive and that it always pushes the DM in the same direction of the WV (i.e. towards a higher level of preservation of environmental resources).

Moreover, in many cases the TV pushes the DM towards preservation of environmental resources more than waiting value (alone) does.

With regard to the level of preservation in the "exogenous and endogenous" information scenario, we find that, with respect to the case in which only exogenous information is available, in many cases (depending on the values of b_1, b_2^u and b_2^f), c_1^* and c_2^* are higher (see $Figure\ B.3$ and $B.4$ as compared to $Figure\ C.1$). This means that in all these cases, accounting for the TV pushes the DM towards a higher level of preservation of the environmental resource.

The reason is that the TV can lead the DM to develop only a certain amount of the environmental asset (*internal solutions*); on the contrary, the WV leads more frequently to *corner solutions*.

Some crucial *Environmental Policy Issues* can be deeply investigated through our "more general" framework. According to the results presented in this chapter, when both exogenous and endogenous information are available,

- it is not obvious that preserving the whole amount of an environmental resource is the optimal choice;

- it is not obvious that the possibility of acquiring new information endogenously leads to develop (destroy) a larger amount of the environmental resource;

- for a large set of values of the benefits of development/preservation, the possibility of endogenous information being acquired and the consequent emergence of the TV leads the DM to destroy less of an environmental resource, with respect to the case in which he takes into account only the WV (allowing only for the possibility of acquiring new information exogenously); hence, for this set of values, "testing" the environmental resource (by destroying a small part of it in $t = 1$) is not only the choice maximizing the DM's intertemporal expected utility, but also the one minimizing the amount of environmental resource destroyed in the current and in the future periods.

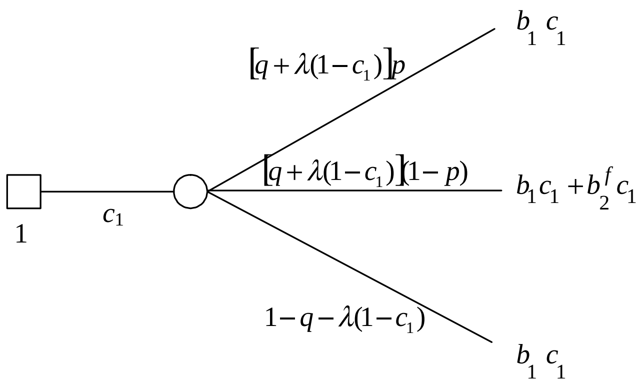
Appendix

Case A: Complete (i.e. Endogenous and Exogenous) Information

Let us solve separately the maximization problem in the two sub-cases:

$$(i) \qquad pb_2^u + (1-p)b_2^f < 0 \qquad \Longrightarrow \qquad c_2{}^* = 0$$

The compound lottery in $Figure$ 2 can be reduced into the one-stage lottery in $Figure$ 3 below.



$Figure$ 3

The expected value of the lottery (given that at $Step$ (c) the DM follows an optimal choice strategy independently from information he has received at Step (b)) is

$$\begin{aligned} EV\left(c_1, (c_2^u)^*, (c_2^f)^*, c_2{}^* = 0\right) &= \{1-(1-p)[q+\lambda(1-c_1)]\} b_1 c_1 + \\ &\quad +(1-p)[q+\lambda(1-c_1)](b_1+b_2^f)c_1 \\ &= b_1 c_1 + (1-p)[q+\lambda(1-c_1)] b_2^f c_1 \end{aligned}$$

By the First Order Condition, we obtain

$$\left.\frac{dEV(c_1)}{dc_1}\right|_{c_2{}^*=0} = b_1 + (1-p)(q+\lambda)b_2^f - 2(1-p)\lambda b_2^f c_1 = 0$$

$$\Longrightarrow \qquad c_1{}^* = \frac{b_1 + (1-p)(q+\lambda)b_2^f}{2(1-p)\lambda b_2^f}$$

Considering that the quantity $\frac{b_1+(1-p)(q+\lambda)b_2^f}{2(1-p)\lambda b_2^f}$ could be lower than 0 or greater than 1, the optimal level of preservation in $t=1$ is

$$
{c_1}^* = \begin{cases} 0 & \text{if} \;\; b_1 \in \left[-\infty, -(1-p)(\lambda+q)b_2^f\right] & (i)' \\ \frac{b_1+(1-p)(q+\lambda)b_2^f}{2(1-p)\lambda b_2^f} & \text{if} \;\; b_1 \in \left[-(1-p)(\lambda+q)b_2^f, (1-p)(\lambda-q)b_2^f\right] & (i)'' \\ 1 & \text{if} \;\; b_1 \in \left[(1-p)(\lambda-q)b_2^f, 0\right] & (i)''' \end{cases}
$$

By sustituting the benefits-dependent optimal levels of c_1 and ${c_2}^* = 0$ in the objective function, we find that the optimal expected value function $EV^*(b_1, b_2^u, b_2^f)$ is

$$
EV^* = \begin{cases} 0 & \text{if} \;\; b_1 \in \left[-\infty, -(1-p)(\lambda+q)b_2^f\right] & (i)' \\ \frac{\left[b_1+(1-p)(q+\lambda)b_2^f\right]^2}{4(1-p)\lambda b_2^f} & \text{if} \;\; b_1 \in \left[-(1-p)(\lambda+q)b_2^f, (1-p)(\lambda-q)b_2^f\right] & (i)'' \\ b_1+(1-p)(q+\lambda)b_2^f & \text{if} \;\; b_1 \in \left[(1-p)(\lambda-q)b_2^f, 0\right] & (i)''' \end{cases}
$$

$$
(ii) \qquad pb_2^u + (1-p)b_2^f > 0 \qquad \Longrightarrow \qquad {c_2}^* = c_1
$$

The compound lottery in *Figure* 2 can be reduced to the one-stage lottery in *Figure* 4 below.

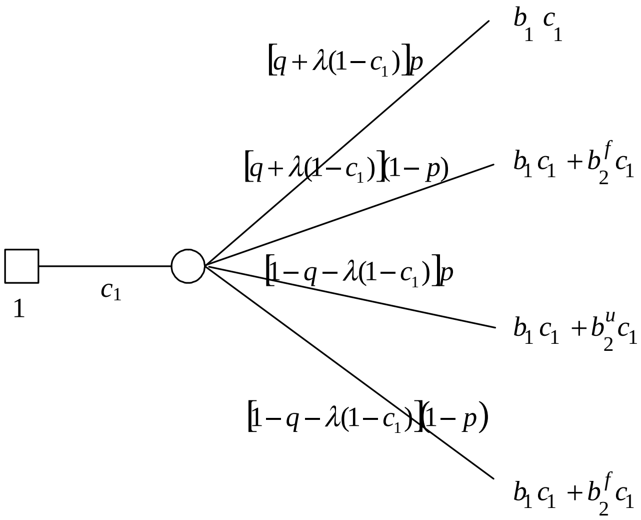
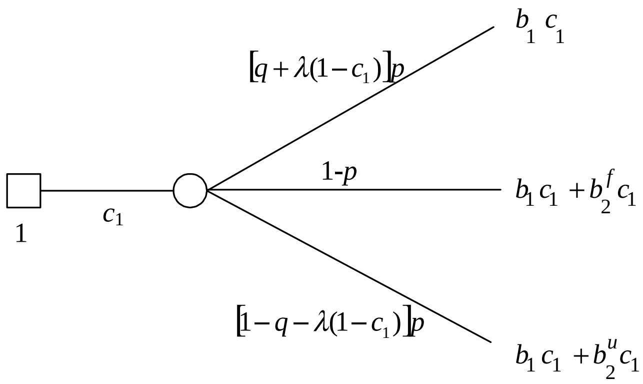


Figure 4

that can be still reduced to the lottery



$Figure$ 5

The expected value of the lottery (given that in $Step$ (c) the DM follows an optimal choice strategy independently from information he has received in Step (b)) is

$$\begin{aligned} EV\left(c_1, (c_2^u)^*, (c_2^f)^*, {c_2}^* = c_1\right) &= b_1c_1 + (1-p)b_2^f c_1 + \{1 - [q + \lambda(1-c_1)]\}\, p b_2^u c_1 \\ &= b_1c_1 + (1-p)b_2^f c_1 + p(1 - q - \lambda + \lambda c_1) b_2^u c_1 \end{aligned}$$

By the First Order Condition, we obtain

$$\begin{aligned} \left.\frac{dEV(c_1)}{dc_1}\right|_{{c_2}^* = c_1} &= b_1 + (1-p)b_2^f + p(1-q-\lambda)b_2^u + 2\lambda p b_2^u c_1 = 0 \\ &\Longrightarrow\ {c_2}^* = -\frac{b_1 + (1-p)b_2^f + p(1-q-\lambda)b_2^u}{2p\lambda b_2^u} \end{aligned}$$

Considering that the quantity $-\frac{b_1+(1-p)b_2^f+p(1-q-\lambda)b_2^u}{2p\lambda b_2^u}$ could be lower than 0 or greater than 1, the optimal level of preservation in $t = 1$ is

$$ {c_1}^* = \begin{cases} 0 & \text{if } \ b_1 + (1-p)b_2^f + (1-q)b_2^u \in [-\infty, \lambda b_2^u] \quad (ii)' \\ -\frac{b_1+(1-p)b_2^f+p(1-q-\lambda)b_2^u}{2p\lambda b_2^u} & \text{if } \ b_1 + (1-p)b_2^f + (1-q)b_2^u \in [\lambda b_2^u, -\lambda b_2^u] \quad (ii)'' \\ 1 & \text{if } \ b_1 + (1-p)b_2^f + (1-q)b_2^u \in [-\lambda b_2^u, +\infty] \quad (ii)''' \end{cases} $$

By sustituting the benefits-dependent optimal levels of c_1 and ${c_2}^* = {c_1}^*$ in the objective function, we find that the optimal expected value function

$EV^*(b_1, b_2^u, b_2^f)$ is

$$EV^* = \begin{cases} 0 & \text{if} \quad b_1 + (1-p)b_2^f + (1-q)b_2^u \in [-\infty, \lambda b_2^u] \quad (ii)' \\ -\frac{\left[b_1+(1-p)b_2^f+p(1-q-\lambda)b_2^u\right]^2}{4p\lambda b_2^u} & \text{if} \quad b_1 + (1-p)b_2^f + (1-q)b_2^u \in [\lambda b_2^u, -\lambda b_2^u] \quad (ii)'' \\ b_1+(1-p)b_2^f+p(1-q)b_2^u & \text{if} \quad b_1 + (1-p)b_2^f + (1-q)b_2^u \in [-\lambda b_2^u, +\infty] \quad (ii)''' \end{cases}.$$

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